Chapter 14: Real Business Cycles

Chapter author: Kurt Mitman

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Introduction and Historical Overview

Introduction

Business cycles: fluctuations of the economy around its long-run trend. The model is "real" because it has no money or nominal variables.

Impulse vs. propagation (Wicksell's rocking horse): shocks are the impulse; the economic structure determines how they propagate.

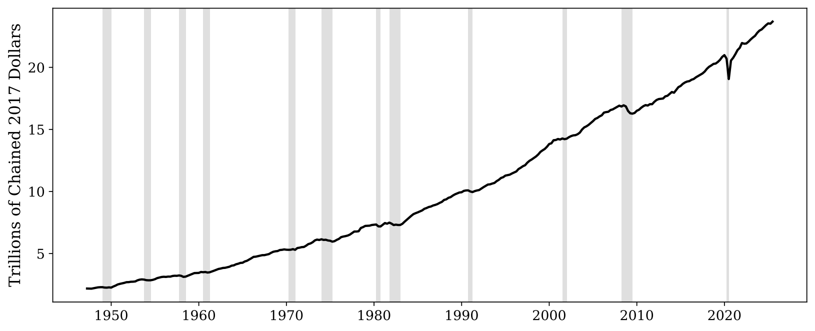
Intellectual history:

- Schumpeter: innovations as impulses
- Keynes: insufficient aggregate demand
- 1970s stagflation $\Rightarrow$ Lucas Critique $\Rightarrow$ microfounded models
- Lucas and Rapping: intertemporal substitution of labor supply
- **Kydland and Prescott (1982)**: operationalized microfounded business cycle analysis $\Rightarrow$ real business cycle (RBC) theory

A First Look at the Data

US Real GDP and Recessions

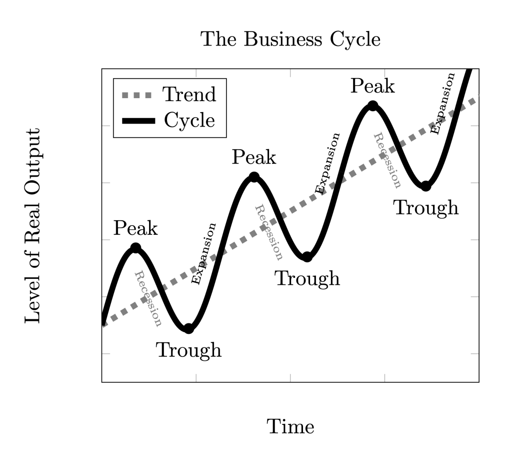
US Real GDP and NBER Recessions, 1949–2022



Strong upward trend; short-run fluctuations barely visible at this scale.

US Real GDP and Recessions

Stylized representation of business cycles



Goal: decompose into trend τ_t and cycle $Y_{c,t}$ with $Y_t = \tau_t + Y_{c,t}$.

The Hodrick-Prescott Filter

Given $\{Y_t\}_{t=1}^{T}$, extract trend $\{\tau_t\}$ by solving:

$$\min_{\{\tau_t\}} \sum_{t=1}^{T} (Y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2$$

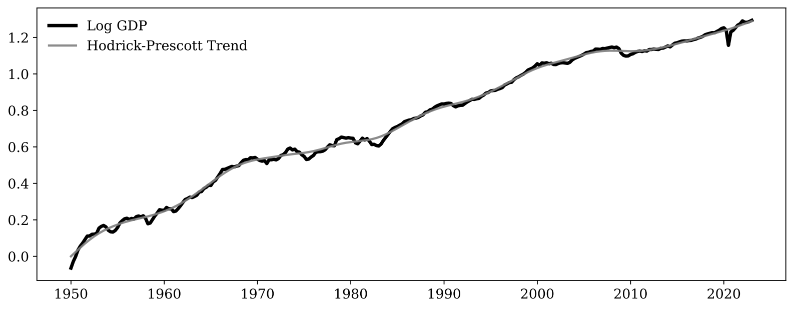
λ controls smoothness: $\lambda \to \infty$ yields a linear trend; $\lambda = 0$ yields $\tau_t = Y_t$.

Standard values: $\lambda = 1{,}600$ (quarterly), $\lambda = 6.25$ (annual) — Ravn and Uhlig (2002): scale by $(1/4)^4$.

Matrix form: $\tau = (I + \lambda A)^{-1} Y$, where A is a pentadiagonal matrix.

The Hodrick-Prescott Filter

Log US GDP: data and HP trend. HP filter with $\lambda = 1{,}600$



Band-Pass Filter

Any stationary process can be represented in the frequency domain:

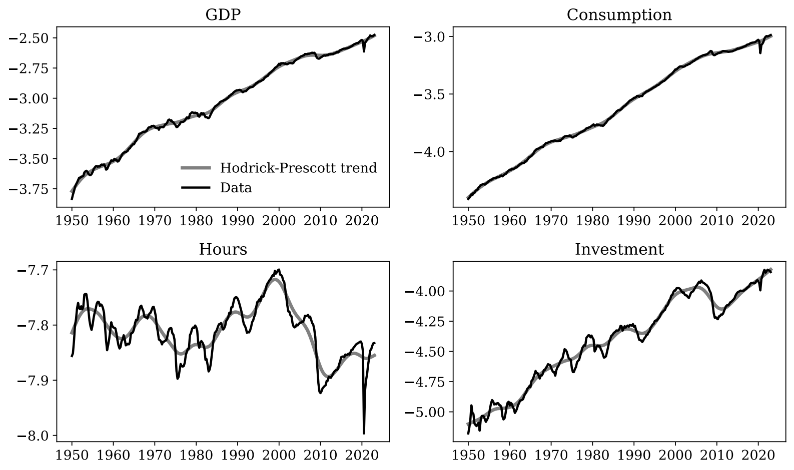
$$Y_t = \int_0^{\pi} A(\omega)\cos(\omega t)\,d\omega + \int_0^{\pi} B(\omega)\sin(\omega t)\,d\omega$$

Band-pass filter: removes frequencies outside a specified range (e.g., 5–10 years for business cycles). Isolates cyclical component directly by frequency.

Complements the HP filter; results are similar for standard choices.

Data and Trends

Actual and HP trends of logs of US aggregates



All variables are per capita, in logs, quarterly, HP-filtered with $\lambda = 1{,}600$.

Stylized Business Cycle Facts

Business cycle moments, US data 1949–2022

Variable x	σ_x (%)	σ_x/σ_y	Autocorr.	Corr(x, y)
Output	1.63	1.00	0.78	1.00
Consumption	1.07	0.65	0.64	0.75
Investment	4.37	2.68	0.86	0.77
Hours	2.11	1.29	0.80	0.86
TFP	0.90	0.55	0.75	0.51
Unemployment	15.63	9.57	0.80	-0.83

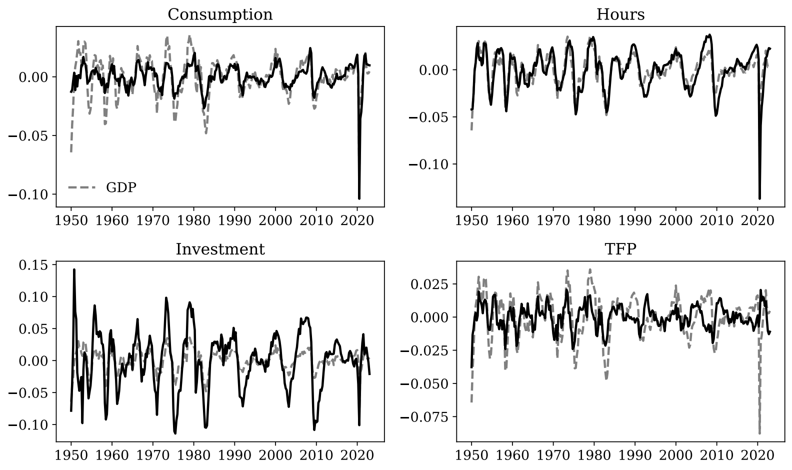
Source: BEA, BLS, FRED. HP-filtered.

Six Stylized Facts

1. **Consumption smoothing**: $\sigma_c/\sigma_y \approx 0.65$
2. **Investment volatility**: $\sigma_i/\sigma_y \approx 2.7$
3. **Labor market**: Hours $\approx$ same volatility as output; unemployment highly volatile and strongly countercyclical
4. **Cyclicality**: C, I, hours, productivity all procyclical; unemployment countercyclical and lagging
5. **Prices and wages**: Much less volatile and less cyclical than quantities
6. **Persistence**: All aggregates highly serially correlated

Deviations from Trend

Deviations from trend of US aggregates (dashed: GDP)



Post-1985: TFP and GDP appear less correlated ⇒ investigate sub-sample stability.

Changing Nature of Business Cycles

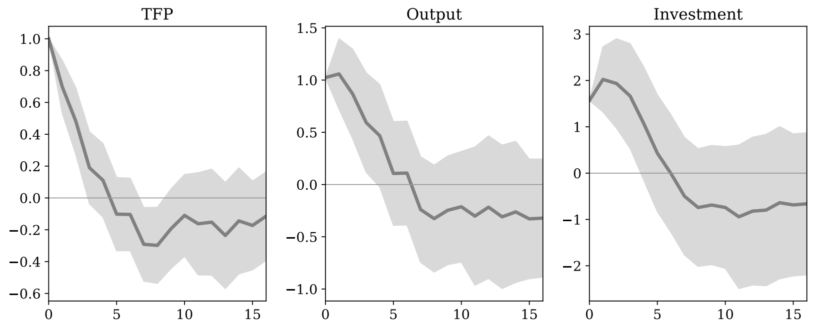
Business cycle moments, US 1985–2022

Variable	σ_x (%)	σ_x/σ_y	Corr(x, y)
Output	1.24	1.00	1.00
Consumption	1.20	0.97	0.86
Investment	3.64	2.94	0.79
Hours	2.19	1.77	0.88
TFP	0.73	0.59	-0.03
Lab. Productivity	1.09	0.88	-0.29

- **Great Moderation**: σ_y falls from 1.97% to 1.24%
- TFP and labor productivity switch from procyclical to acyclical/countercyclical
- "Jobless recoveries" post-1991

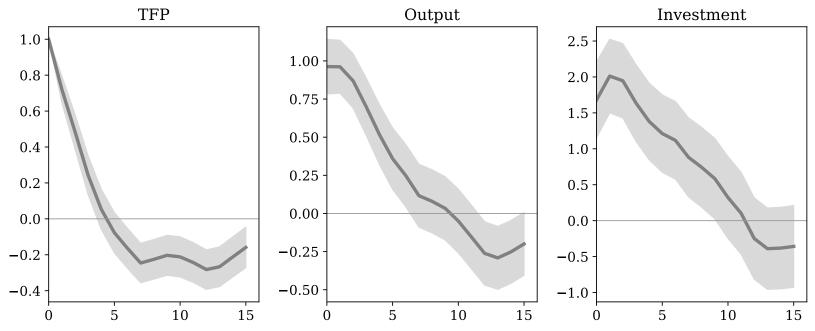
Conditional Moments: Estimated IRFs

IRFs to TFP shock estimated with structural VAR (Recursive identification; TFP ordered first)



Conditional Moments: Estimated IRFs

IRFs to TFP shock estimated with local projections



The Simple RBC Model (No Capital)

Setup

Preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [\log(c_t) + \phi \log(1 - \ell_t)]$$

Production: (linear, no capital)

$$Y_t = z_t \ell_t$$

TFP:

$$\log z_t = \rho \log z_{t-1} + \sigma \varepsilon_t$$

Budget: $c_t + B_t = w_t \ell_t + R_t B_{t-1}$, bonds in zero net supply.

Intratemporal FOC: $w_t / c_t = \phi / (1 - \ell_t)$

Euler equation: $1/c_t = \beta \mathbb{E}_t [R_{t+1} / c_{t+1}]$

Constant Labor Supply

In equilibrium $B_t = 0$ and $c_t = w_t \ell_t = z_t \ell_t$. Substituting into the intratemporal FOC:

$$\frac{w_t}{w_t \ell_t} = \frac{1}{\ell_t} = \frac{\phi}{1 - \ell_t} \quad \Rightarrow \quad \ell_t = \frac{1}{1 + \phi} \quad \text{(constant)}$$

Output, consumption, and TFP are perfectly correlated; hours do not fluctuate.

Why? Income and substitution effects from balanced-growth preferences exactly cancel. Without savings, the interest rate adjusts to make intertemporal substitution zero.

Lesson: TFP shocks + elastic labor supply alone are not sufficient for business cycles. Need capital accumulation.

The Core RBC Model

Setup

Preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - \ell_t)$$

Production: (CRS, neoclassical)

$$Y_t = z_t F(k_t, \ell_t)$$

TFP:

$$\log z_t = \rho_z \log z_{t-1} + \sigma_z \varepsilon_t$$

Capital accumulation: $k_{t+1} = (1 - \delta) k_t + i_t$

Resource constraint: $c_t + i_t = Y_t$

Planning problem (recursive, CE is efficient):

$$V(k, z) = \max_{k', \ell} \left\{ U(zF(k, \ell) + (1 - \delta)k - k', 1 - \ell) + \beta \mathbb{E}[V(k', z') | z] \right\}$$

Key Optimality Conditions

Intratemporal (labor-leisure):

$$zF_2(k, \ell) = \frac{U_2(c, 1-\ell)}{U_1(c, 1-\ell)}$$

Intertemporal (Euler equation):

$$U_1(c, 1-\ell) = \beta \mathbb{E}\left[(z' F_1(k', \ell') + 1 - \delta) U_1(c', 1-\ell') \,\middle|\, z\right]$$

- Positive TFP shock $\Rightarrow$ MPL rises $\Rightarrow$ labor supply increases (Frisch elasticity)
- Persistent shock $\Rightarrow$ MPK rises tomorrow $\Rightarrow$ investment increases (EIS)
- Capital accumulation breaks the perfect consumption-output correlation

Calibration (Kydland-Prescott Blueprint)

Functional forms: $U = (1-\phi)\log c + \phi \log(1-\ell)$, $F = k^\alpha \ell^{1-\alpha}$

Parameters:

- $\alpha = 0.36$ (capital share from NIPA)
- $\delta = 0.025$ (quarterly depreciation from perpetual inventory)
- $\beta = 0.99$ (4% annual discount rate, from $\sigma = 1$ and steady-state Euler)
- $\phi = 0.6325$ (targets $\bar{\ell} = 1/3$ using intratemporal FOC and $\bar{y}/\bar{c}$)
- $\rho_z = 0.95$, $\sigma_z = 0.007$ (estimated from Solow residual)

TFP measured via growth accounting:
$\log z_t = \log y_t - \alpha \log k_t - (1-\alpha)\log \ell_t$.

Model vs. Data

Business cycle moments in core RBC model

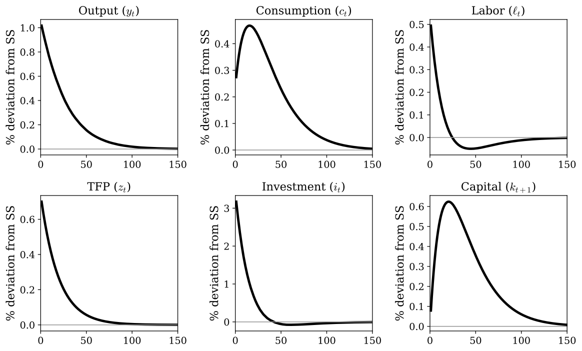
Variable	σ_x (%)	σ_x/σ_y	Autocorr.	Corr(x, y)
Output	1.33	1.00	0.73	1.00
Consumption	0.42	0.32	0.81	0.90
Investment	4.13	3.10	0.72	0.99
Hours	0.64	0.48	0.72	0.98
TFP	0.92	0.69	0.43	0.99

Successes: C smooth, I volatile, all variables procyclical.

Shortcomings: Output volatility too low (1.33 vs. 1.97); hours too smooth ($\sigma_h/\sigma_y = 0.48$ vs. 1.29); correlations too high (single shock).

Impulse Response Functions

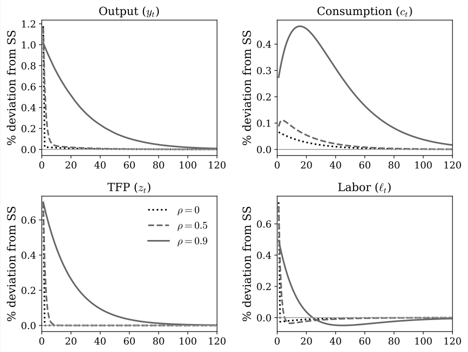
TFP shock in RBC model. $\rho_z = 0.95$



- On impact: y jumps (TFP $+ \uparrow \ell$); c rises less ($\Rightarrow$ saving); i overshoots
- k builds up gradually, peaks after TFP has largely faded
- **Amplification**: $\sigma_y > \sigma_z$ (hours amplify output response)
- **Propagation**: Limited; output persistence $\approx$ TFP persistence

Sensitivity to Persistence

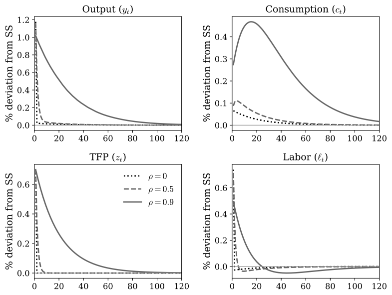
Sensitivity to TFP shock persistence ($\rho = 0, 0.5, 0.95$)

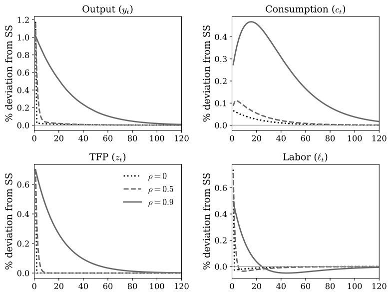


- Less persistent shocks: **more** impact amplification (households willing to substitute more intertemporally)
- Output persistence tracks TFP persistence closely
- The model does not generate endogenous propagation beyond TFP itself

Sensitivity to EIS and Frisch Elasticity

Sensitivity to EIS ($1/\sigma$) and Frisch elasticity





- Higher Frisch $\Rightarrow$ more amplification (hours respond more), but investment becomes too volatile
- Higher EIS $\Rightarrow$ more amplification; consumption response non-monotonic on impact
- Trade-off: improving hours volatility worsens investment volatility

Extensions

Indivisible Labor

Hansen (1985), Rogerson (1988): Workers either work $\bar{\ell}$ or zero (extensive margin).

With lotteries to convexify: aggregate Frisch elasticity becomes **infinite** (linear disutility), resolving the low micro-Frisch elasticity problem.

Generates larger fluctuations in employment, consistent with the data showing most hours variation occurs at the extensive margin.

Capital Adjustment Costs

Investment adjustment cost modifies capital accumulation:

$$k_{t+1} = (1-\delta)k_t + i_t - \frac{\psi}{2}\left(\frac{i_t}{k_t} - \delta\right)^2 k_t$$

Shadow price of installed capital (Tobin's q):

$$q_t = 1 + \psi\left(\frac{i_t}{k_t} - \delta\right)$$

Firms spread investment over time $\Rightarrow$ dampens σ_i, increases persistence. Also used in New Keynesian models.

Variable Capacity Utilization

Greenwood, Hercowitz, and Huffman (1988): Effective capital $= u_t k_t$, with higher utilization causing faster depreciation:

$$k_{t+1} = (1 - \delta(u_t))k_t + i_t$$

Additional adjustment margin $\Rightarrow$ more amplification. But Solow residual must be corrected for variable utilization.

Additional Shocks

Investment-specific technology shocks (Greenwood, Hercowitz, Krusell, 1997):

$$k_{t+1} = (1-\delta)k_t + q_t i_t$$

Account for $\sim$30% of business cycle fluctuations. Break the strong c-i correlation.

Government spending shocks: $c_t + i_t + g_t = y_t$. Negative income effect from $g \Rightarrow$ increases labor supply. Breaks the high hours-wage correlation.

Business Cycle Accounting

The BCA Framework

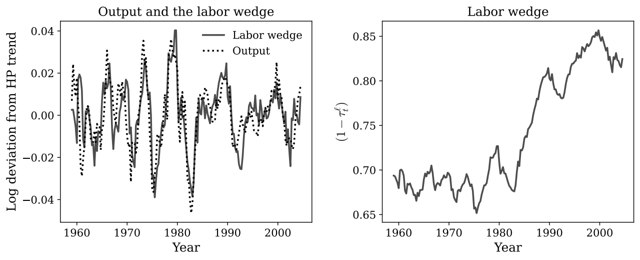
Chari, Kehoe, and McGrattan (2007): Introduce four **wedges** into the RBC model, measured as deviations from first-best conditions:

1. **Efficiency wedge** (z_t): TFP in $Y_t = z_t F(k_t, \ell_t)$
2. **Labor wedge** $(1 - \tau_t^\ell)$: $(1 - \tau_t^\ell) z_t F_2(k_t, \ell_t) = U_2/U_1$
3. **Investment wedge** $(1/(1 + \tau_t^i))$: modifies the Euler equation
4. **Government spending wedge** (g_t): $c_t + i_t + g_t = Y_t$

Estimate AR(1) processes for wedges via maximum likelihood on (y_t, ℓ_t, i_t, g_t), then simulate each wedge in isolation.

BCA Results

The estimated labor wedge from BCA



- Efficiency wedge: ∼70% of output variance
- Labor wedge: ∼25% of output variance; explains ∼75% of hours variance
- Investment and government wedges: minor
- Labor wedge highly procyclical
- Secular improvement in labor wedge coincides with changing business cycle patterns post-1985

Summary

Key Takeaways (I)

1. **Business cycle facts**: C smooth, I volatile, hours procyclical, unemployment countercyclical. All aggregates highly persistent. Post-1985: Great Moderation, TFP becomes acyclical, jobless recoveries.
2. **HP filter**: $\min \sum (Y_t - \tau_t)^2 + \lambda \sum [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2$; $\lambda = 1{,}600$ quarterly.
3. **Simple model** (no capital): Balanced-growth preferences $\Rightarrow$ constant hours. Income and substitution effects cancel. Capital accumulation is essential.
4. **Core RBC**: Stochastic NGM; intratemporal FOC (MPL $=$ MRS) and Euler equation. Capital provides savings channel $\Rightarrow$ hours fluctuate.

Key Takeaways (II)

5. **Calibration**: α, δ, β from long-run data; ϕ targets $\bar{\ell} = 1/3$; (ρ_z, σ_z) from Solow residual. Model explains ∼70% of output variance.
6. **Shortcomings**: Hours too smooth; correlations too high (single shock); limited endogenous propagation.
7. **Extensions**: Indivisible labor (infinite aggregate Frisch), adjustment costs (dampen σ_i, Tobin's q), variable utilization (more amplification), additional shocks (IST, government).
8. **Business cycle accounting**: Efficiency wedge (∼70%) and labor wedge (∼25%) dominate. Labor wedge points toward frictional labor markets (Chapter 20) and heterogeneity (Chapter 21).