Chapter 11: Consumption

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Outline

1. Introduction
2. Autarky and Full Insurance
3. Income Fluctuation Problems
4. Heterogeneous-Agent Incomplete-Market Models
5. Summary

Introduction

Motivation

Household consumption is a key determinant of welfare:

- Growth of per-capita consumption = rising prosperity
- Consumption fluctuations are costly $\Rightarrow$ stabilization policy
- Consumption distribution measures inequality in living standards better than income
- Consumption choices determine stochastic discount factors $\Rightarrow$ asset prices

Roadmap

1. Two extreme market structures: autarky vs. complete markets; empirical tests
2. The income fluctuation problem (bond economy): PIH, borrowing constraints, precautionary saving
3. Heterogeneous-agent incomplete-market equilibrium (Huggett, Bewley, Aiyagari)

The organizing question throughout: *how much of an idiosyncratic income shock passes through to consumption?* Autarky says all of it, complete markets say none of it, and the truth lies in between.

Autarky and Full Insurance

Setup

Endowment economy with aggregate uncertainty. Histories $\omega^t = \{\omega_0, \ldots, \omega_t\} \in \Omega^t$ with probability $\pi(\omega^t)$. Continuum of households i with stochastic income $y_{i,t}(\omega^t)$ such that

$$\int_0^1 y_{i,t}(\omega^t)\, di = Y_t(\omega^t).$$

Expected utility with $u' > 0$, $u'' < 0$. Aggregate feasibility: $C_t(\omega^t) = Y_t(\omega^t)$ for all t, ω^t.

Autarky: no insurance markets, no storage. Budget constraint:

$$c_{i,t}(\omega^t) = y_{i,t}(\omega^t) \quad \text{for all } t, \omega^t.$$

Full pass-through of income shocks to consumption: no smoothing whatsoever.

Complete Markets: Full Risk Sharing

Households trade a complete set of Arrow securities subject to the time-zero budget:

$$\sum_{t=0}^{\infty} \sum_{\omega^t \in \Omega^t} p_t(\omega^t) \left[c_{i,t}(\omega^t) - y_{i,t}(\omega^t) \right] = 0.$$

Equilibrium characterization (from Chapter 7): for any pair (i, j),

$$\frac{u'(c_{i,t}(\omega^t))}{u'(c_{j,t}(\omega^t))} = \frac{\lambda_i}{\lambda_j} \quad \text{for all } t, \omega^t.$$

The ratio of marginal utilities is **constant across time and states**.

Complete Markets: The CRRA Case

With CRRA preferences and common curvature σ, summing over households and using aggregate feasibility:

$$c_{j,t}(\omega^t) = \left[\frac{(\lambda_j)^{-1/\sigma}}{\int_0^1 (\lambda_i)^{-1/\sigma} di} \right] C_t(\omega^t).$$

Individual consumption is **proportional to aggregate consumption**:

- Independent of own income realization
- Only uninsurable *aggregate* shocks move individual consumption
- The constant of proportionality reflects initial wealth through λ_j

Full Insurance with Preference Heterogeneity

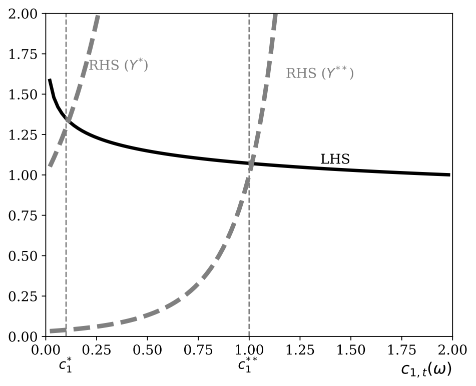
Two agents with CRRA curvatures $\sigma_1 < \sigma_2$. FOCs and market clearing imply:

$$\lambda_2 c_{1,t}(\omega^t)^{-\sigma_1} = \lambda_1 \left[Y_t(\omega^t) - c_{1,t}(\omega^t)\right]^{-\sigma_2}.$$

Marginal utility ratios are still equalized, but consumption *ratios* are not: it is efficient for the **least risk-averse** agents to absorb aggregate fluctuations.

Full Insurance: Determination of $c_{1,t}$

LHS and RHS as functions of $c_{1,t}$



- $\sigma_1 = 0.1$, $\sigma_2 = 5$, $\lambda_1 = \lambda_2 = 1$; endowment rises from $Y^* = 1$ to $Y^{**} = 2$

Empirical Tests of Full Insurance

Encompassing regression:

$$\Delta \log c_{i,t} = \beta_1 \Delta \log C_t + \beta_2 \Delta \log y_{i,t} + \varepsilon_{i,t}.$$

- **Autarky**: $\beta_1 = 0$, $\beta_2 = 1$ (consumption tracks own income)
- **Full insurance**: $\beta_1 = 1$, $\beta_2 = 0$ (aggregate only)

Evidence: Mace (1991) rejects $\beta_1 = 1$; Cochrane (1991) rejects $\beta_2 = 0$ using PSID income-loss events; Attanasio and Davis (1996): "spectacular failure" across education/cohort groups.

Conclusion: reality lies between the two benchmarks — **partial insurance**.

Two Approaches to Partial Risk Sharing

Endogenous incomplete markets: model the frictions (limited commitment, private information) that prevent full insurance; the market structure emerges in equilibrium.

- Strength: structure responds to primitives and policy
- Weakness: equilibrium contracts are complex and counterfactual (e.g., extreme left-skewness of consumption, Broer 2013)

Exogenous incomplete markets: model only the assets observed in reality — the **bond economy**.

- Strength: easily taken to the data
- Weakness: asset structure does not respond to primitives

The Bond Economy

Complete-markets sequential budget, with $a_{i,t+1}(\omega_{t+1}, \omega^t)$ a full set of state-contingent claims:

$$c_{i,t} + \sum_{\omega_{t+1} \in \Omega} q_t(\omega_{t+1}, \omega^t) a_{i,t+1}(\omega_{t+1}, \omega^t) = y_{i,t} + a_{i,t}.$$

Restrict trade to a single **non-state-contingent bond**:

$$c_{i,t}(\omega^t) + q_t(\omega^t) a_{i,t+1}(\omega^t) = y_{i,t}(\omega^t) + a_{i,t}(\omega^{t-1}).$$

With a constant aggregate endowment, $q_t = q$ and $r = 1/q - 1$:

$$a_{i,t+1} = (1 + r)(y_{i,t} + a_{i,t} - c_{i,t}).$$

Saving and dissaving act as **self-insurance**: trading intertemporally with oneself.

Income Fluctuation Problems

The Deterministic Problem

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \quad \text{s.t.} \quad a_{t+1} = (1+r)(y_t + a_t - c_t).$$

Euler equation:

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = 1 + r, \qquad \text{and with CRRA} \qquad \frac{c_{t+1}}{c_t} = [\beta(1+r)]^{1/\sigma}.$$

The slope of the consumption path is increasing in β and r; the strength of the response is governed by the EIS $= 1/\sigma$. Households save when income is high relative to its mean and dissave when it is low.

Consumption Function and MPC

Iterate the budget constraint forward (using the TVC) and substitute the Euler equation, with $R \equiv 1 + r$:

$$c_t = \left[1 - R^{-1}(\beta R)^{1/\sigma}\right] \left[a_t + \sum_{j=0}^{\infty} \left(\frac{1}{R}\right)^j y_{t+j}\right].$$

Consumption is **linear** in total wealth = financial wealth + human wealth (Friedman's permanent income hypothesis).

Marginal propensity to consume:

$$MPC = 1 - R^{-1}(\beta R)^{1/\sigma}.$$

MPC: Special Cases and Permanent Income

- If $\beta R = 1$: $MPC = 1 - \beta \approx \rho$, the discount rate
- If $\sigma = 1$ (log utility): $MPC \approx r$

MPC out of a permanent income change: raise y_{t+j} by 1 for all $j \geq 0$ with $\beta R = 1$. Human wealth rises by $1/(1 - R^{-1})$, so consumption rises by exactly **1**.

Transitory and permanent MPCs differ sharply — $1 - R^{-1}$ versus 1. This is one of the key insights of Friedman's theory of consumption.

Permanent Income Hypothesis: Hall's Martingale

Stochastic income, **quadratic utility** $u(c) = b_1 c - \frac{1}{2} b_2 c^2$, and $\beta R = 1$. The Euler equation becomes

$$c_t = \mathbb{E}_t c_{t+1}.$$

Consumption is a martingale (Hall, 1978). By the law of iterated expectations, $\mathbb{E}_t c_{t+j} = c_t$ for all $j \geq 0$: the best forecast of all future consumption is today's consumption.

Testable implication: no variable dated t or earlier should help predict Δc_{t+1}.

PIH: Annuity Value and Certainty Equivalence

Iterating the budget constraint and using the martingale property:

$$c_t = \frac{r}{1+r}\left[a_t + \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j \mathbb{E}_t y_{t+j}\right].$$

Consumption equals the **annuity value of total wealth**.

Certainty equivalence: only the conditional *mean* of future income matters — no higher moment of the income process affects consumption. This is a consequence of the linear-quadratic structure, and it rules out precautionary saving by construction.

Consumption Responds to News

The change in consumption equals the annuitized innovation in permanent income:

$$\Delta c_t = \frac{r}{1+r} \sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^j (\mathbb{E}_t - \mathbb{E}_{t-1}) y_{t+j}.$$

Only **news** arriving between $t-1$ and t moves consumption. Anticipated income changes have no effect.

Transitory versus Permanent Shocks

Permanent-transitory income process (Abowd-Card):

$$y_t = y_t^p + u_t, \qquad y_t^p = y_{t-1}^p + v_t,$$

with u_t and v_t orthogonal i.i.d. shocks. Then

$$\Delta c_t = \frac{r}{1+r} u_t + v_t.$$

Households consume the **annuity value of transitory** shocks and the **full value of permanent** shocks.

$\Rightarrow$ The bond economy is close to full insurance against transitory shocks, but offers essentially no insurance against permanent ones.

Application: Inequality Decomposition

Blundell and Preston (1998): with $r \approx 0$, within-cohort moments satisfy

$$\Delta \mathsf{Var}_{k,t}(c) = \Delta \mathsf{Cov}_{k,t}(c, y) \approx \mathsf{Var}_t(v).$$

The change in **consumption** inequality identifies the variance of **permanent** income shocks — consumption data reveal the nature of income risk.

Finding: the bulk of the rise in UK income inequality over the 1980s and 1990s was permanent rather than transitory.

Application: Saving for a Rainy Day

Campbell (1987): define saving as

$$s_t = \frac{r}{1+r} a_t + y_t - c_t.$$

Substituting the consumption function:

$$s_t = -\sum_{j=1}^{\infty} \left(\frac{1}{1+r} \right)^j \mathbb{E}_t \Delta y_{t+j}.$$

Savings equal the discounted sum of expected income **declines**: households save in anticipation of rainy days and dissave when they expect sunny days.

Borrowing Constraints: When Do They Matter?

Impose $a_{t+1} \geq 0$. Whether it binds depends on the income process:

- y_t a **random walk**: $c_t = \frac{r}{1+r} a_t + y_t$, so $a_{t+1} = a_t$ is constant — the constraint never binds if $a_0 > 0$
- y_t **i.i.d.**: wealth follows a driftless random walk, so the constraint binds in finite time with positive probability

The same preferences can therefore deliver very different behavior depending on the persistence of income.

The Modified Euler Equation

With multiplier $\lambda_t \geq 0$ on the constraint:

$$u'(c_t) \geq \beta R \mathbb{E}_t[u'(c_{t+1})],$$

holding with strict inequality when the constraint binds. In that case the household consumes all cash in hand, $c_t = a_t + y_t$.

Two key differences versus the unconstrained problem:

1. The **timing** of income matters, not just its present value
2. Constrained households have $MPC = 1$ out of *both* transitory and permanent shocks

The Natural Borrowing Limit

The lowest asset position consistent with non-negative consumption in every state. With lowest income realization y_{min} occurring with positive probability:

$$a_{t+1} \geq -\left(\frac{1+r}{r}\right) y_{min}.$$

Borrowing to this limit and then drawing $y_t = y_{min}$ forever forces $c_t = 0$.

$\Rightarrow$ If u satisfies the Inada condition $u(0) = -\infty$, the natural limit **never binds**: the Euler equation always holds with equality.

Ad hoc limits tighter than the natural one *do* bind. If $y_{min} = 0$, the no-borrowing constraint coincides with the natural limit.

Precautionary Saving I: Borrowing Constraints

Quadratic utility (no prudence, $u''' = 0$), $\beta R = 1$, limit $a_{t+1} \geq -\underline{a}$:

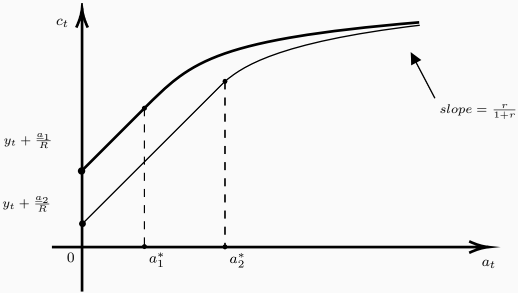
$$c_t = \min\left\{y_t + a_t + \frac{\underline{a}}{R},\ \mathbb{E}_t c_{t+1}\right\}.$$

A mean-preserving spread in y_{t+1} makes low realizations — and hence a binding constraint — more likely. The expectation of the min falls, so c_t falls and saving rises.

Precautionary saving arises here *without* any prudence in preferences: the constraint alone generates it.

The Consumption Function with a Constraint

Decision rule for consumption with a borrowing constraint

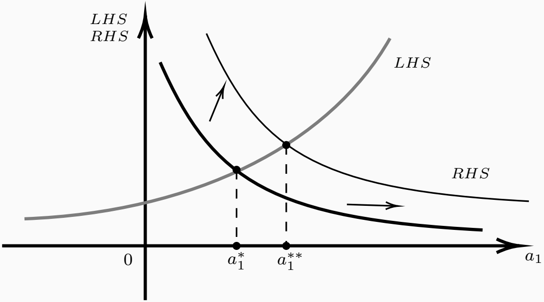


- Slope 1 below a^*, concave above, asymptoting to $r/(1+r)$
- The two curves are different borrowing limits: tightening shifts consumption down and steepens it
- $MPC \in (r/(1+r), 1)$ — **above the PIH value**, larger near the constraint

Precautionary Saving II: Prudence

Two-period problem (Leland, 1968; Sandmo, 1970):

$u'(y_0 - a_1) = \beta R \mathbb{E}_0[u'(Ra_1 + y_1)]$



- A mean-preserving spread in y_1 shifts the RHS up when u' is convex, raising a_1
- **Prudence**: $u''' > 0$ (u' convex) — by Jensen, risk raises $\mathbb{E}_0 u'$

Measuring Prudence

Kimball (1990) coefficients:

$$\text{absolute prudence} = -\frac{u'''(c)}{u''(c)}, \qquad \text{relative prudence} = -\frac{u'''(c)c}{u''(c)}.$$

DARA implies prudence: if absolute risk aversion $\alpha(c)$ is decreasing, $\alpha'(c) < 0$, then

$$u'''(c) > \frac{[u''(c)]^2}{u'(c)} > 0.$$

CRRA utility is DARA, and hence prudent. Quadratic utility is not: $u''' = 0$.

The Role of Prudence: Analytical Expression

Blanchard and Mankiw (1988): a second-order approximation of the Euler equation yields

$$\mathbb{E}_t\left[\frac{c_{t+1}-c_t}{c_t}\right] \approx EIS(c_t)(r-\rho) + \frac{1}{2}P(c_t)\mathbb{E}_t\left[\left(\frac{c_{t+1}-c_t}{c_t}\right)^2\right],$$

where

$$EIS(c_t) = -\frac{u'(c_t)}{c_t u''(c_t)}, \qquad P(c_t) = -\frac{u'''(c_t)c_t}{u''(c_t)}.$$

Expected consumption growth has two parts: intertemporal smoothing through $r-\rho$, and a **precautionary term** proportional to relative prudence times consumption growth variability.

Concavity of the Consumption Function

Carroll and Kimball (1996): under HARA utility with $u''' > 0$, the consumption function $c(a)$ is concave — strictly concave in the CRRA case.

Two knife-edge exceptions within HARA:

- **CARA** ($\kappa = 1$): consumption function is linear
- **Quadratic** ($\kappa = 0$): consumption function is linear

Concavity is what delivers a declining MPC in wealth — the property that makes the distribution of wealth matter for aggregate consumption dynamics.

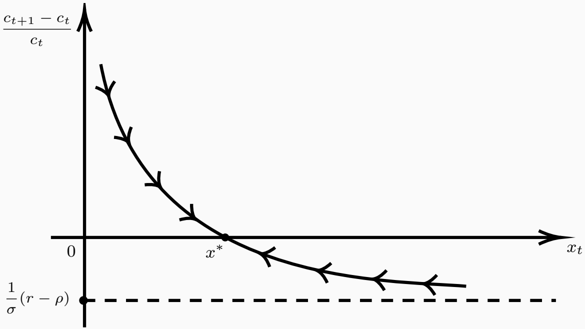
Carroll (1992, 2001): CRRA utility, log income = permanent + transitory shocks, positive probability of zero income (so the natural limit is 0), and **impatience**, $\rho > r$.

Single state variable: $x_t =$ cash in hand relative to permanent income. Consumption growth satisfies

$$\mathbb{E}_t \Delta \log c_{t+1} \approx \frac{1}{\sigma}(r - \rho) + \frac{\sigma + 1}{2} \mathsf{Var}_t(\Delta \log c_{t+1}).$$

Impatience pushes consumption growth down; risk pushes it up.

Consumption dynamics as a function of cash in hand x



- Above x^* impatience dominates and households dissave; below x^* they save
- Wealth is **stationary** at a target buffer above the certainty-equivalent level

Excess Sensitivity and Liquidity Constraints

PIH prediction: Δc_t responds only to date-t news, so past income changes should not matter. This is empirically rejected — **excess sensitivity** (Flavin, 1981; Campbell and Mankiw, 1989).

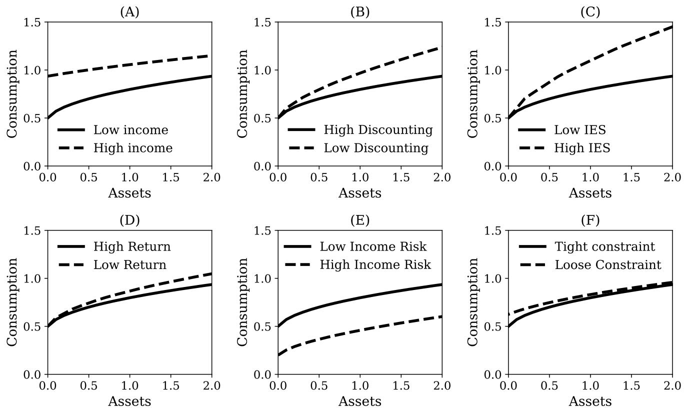
Zeldes (1989): with a borrowing limit the log-linearized Euler equation becomes

$$\Delta \log c_{t+1} = \frac{1}{\sigma}(r-\delta)+\frac{\sigma+1}{2}\mathsf{Var}_t(\Delta \log c_{t+1})+\frac{1}{\sigma}\log(1+\lambda_t)+\varepsilon_{t+1}.$$

The unobserved multiplier λ_t is an omitted variable: high lagged income relaxes the constraint, generating a negative coefficient on lagged income for constrained households. Splitting the PSID by wealth confirms that constraints bind for the **low-wealth** subgroup only.

Comparative Statics of the Consumption Function

Comparative statics of the optimal consumption function



CRRA $\sigma = 2$, $\beta = 0.95$, $R = 1.02$, zero borrowing limit, two-state Markov income $\{0.5, 1.5\}$ with persistence 0.8.

Comparative Statics: Panels (A)–(C)

(A) **Income**: low-y households consume less — fewer resources and a stronger precautionary motive; slopes converge at high wealth

(B) **Discounting**: impatient households consume more, with a higher MPC

(C) **IES**: with $\beta R < 1$, high-IES households consume more

Comparative Statics (continued)

(D) **Return**: a higher R induces more accumulation and less consumption at any given wealth level

(E) **Income risk**: a mean-preserving spread lowers consumption at every wealth level — more precautionary saving

(F) **Credit limit**: a looser constraint reduces precautionary saving, since borrowing capacity substitutes for buffer wealth

Panels (E) and (F) make the same point from two directions: what matters is the household's total capacity to absorb a bad shock, whether through accumulated assets or through access to credit.

Bounds on Wealth Accumulation

For equilibrium existence we need a bounded asset space.

Sufficient condition: $\beta R < 1$, with i.i.d. shocks and DARA utility (Schechtman and Escudero, 1977).

Sketch: let x denote cash in hand and $\bar{x}' = Ra'(x) + \bar{y}$ the best-case cash next period. The Euler equation gives

$$u_c(c(x)) = \beta R \frac{\mathbb{E}[u_c(c(x'))]}{u_c(c(\bar{x}'))} u_c(c(\bar{x}')).$$

Bounds on Wealth: Completing the Argument

A Taylor expansion of the ratio gives

$$\frac{\mathbb{E}[u_c(c(x'))]}{u_c(c(\bar{x}'))} \approx 1 + \alpha(c(\bar{x}'))[\bar{y} - \mathbb{E}(y')]c_x(\bar{x}').$$

If $\lim_{x\to\infty} \alpha(c(x)) = 0$, which DARA delivers, the ratio tends to 1. Hence for large x,

$$u_c(c(x)) < u_c(c(\bar{x}')) \quad \Rightarrow \quad \bar{x}' < x.$$

Cash in hand cannot grow forever: as wealth rises the precautionary motive vanishes and impatience ($\beta R < 1$) takes over.

Heterogeneous-Agent Incomplete-Market Models

The Huggett Endowment Economy

Continuum of ex-ante identical households with idiosyncratic endowment $y_t \in Y$ (finite), following a first-order ergodic Markov chain $\pi(y'|y)$ with invariant distribution $\Pi^*(y)$. No aggregate uncertainty.

Recursive household problem:

$$V(a, y) = \max_{c, a'} \left\{ u(c) + \beta \sum_{y' \in Y} \pi(y'|y) V(a', y') \right\}$$

subject to

$$c + a' = Ra + y, \qquad a' \geq -\underline{a}.$$

The asset is in **zero net supply**: pure borrowing and lending among households.

State Space and Transition Function

State space $S = A \times Y$ with $A = [-\underline{a}, \bar{a}]$, and a distribution λ over (a, y).

The transition function for any measurable set $\mathcal{A} \times \mathcal{Y}$:

$$Q((a, y), \mathcal{A} \times \mathcal{Y}) = \mathbb{I}_{\{a'(a,y) \in \mathcal{A}\}} \sum_{y' \in \mathcal{Y}} \pi(y'|y).$$

The indicator reflects the deterministic asset choice given the state; the sum reflects the stochastic income transition. Together they induce a Markov process on the joint distribution of assets and income.

Stationary Equilibrium (Huggett)

A recursive stationary competitive equilibrium is a value function $V : S \to \mathbb{R}$, policy functions a' and c, an interest rate r^*, and a stationary measure λ^* such that:

1. Given r, the policies a' and c solve the household problem with value function V
2. The asset market clears at r^*:
$$\int_{A\times Y} a'(a,y)\,d\lambda^* = 0.$$
3. The goods market clears: $\int_{A\times Y} c(a,y)\,d\lambda^* = \sum_{i=1}^{N} y_i \Pi^*(y_i)$

Stationary Equilibrium: The Invariant Measure

4. For all $(\mathcal{A} \times \mathcal{Y}) \in \Sigma_s$, the measure reproduces itself:

$$\lambda^*(\mathcal{A} \times \mathcal{Y}) = \int_{A \times Y} Q((a, y), \mathcal{A} \times \mathcal{Y})\, d\lambda^*.$$

The key conceptual point: the *distribution* is constant over time, but *individual households move within it*. There is continuous churning — households save up, are hit by bad shocks, run down assets — with no change in the cross-sectional distribution.

Existence of Equilibrium

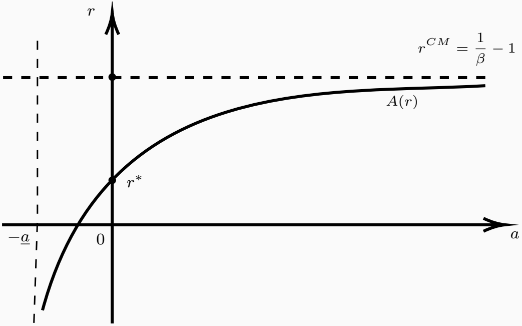
Define excess asset demand $A(r) = \int_{A\times Y} a'(a, y; r)\, d\lambda_r^*$.

- **Continuity**: theorem of the maximum makes a' continuous in r; Stokey-Lucas-Prescott conditions give existence and uniqueness of λ_r^* (compact state space plus a monotone mixing condition)
- As $r \to 1/\beta - 1$: $A(r) \to +\infty$, since accumulation is unbounded when $\beta R = 1$
- At $r = -1$: $A(-1) = -\underline{a}$, everyone borrows to the limit

By continuity, $A(r)$ crosses zero at some $r^* \in (-1, 1/\beta - 1)$.

Equilibrium of the Endowment Economy

Equilibrium of the endowment economy with zero net supply



- $A(r)$ crosses zero at $r^* < r^{CM} = 1/\beta - 1$, so $\beta R^* < 1$ as assumed
- **Uniqueness** needs $A(r)$ monotone; $\sigma < 1$ suffices (Achdou et al., 2022)

The Risk-Free Rate Puzzle

In the complete-markets representative-agent model, a high equity premium requires high risk aversion, hence a low EIS, hence a strong desire to borrow against growing income — and therefore a counterfactually **high risk-free rate** (Mehra and Prescott; Weil).

Huggett (1993): with uninsurable idiosyncratic risk, the precautionary saving motive pushes the equilibrium risk-free rate **down**, toward the value observed in the data.

Incomplete markets thus help resolve the risk-free rate puzzle — an equilibrium payoff from taking household heterogeneity seriously.

Assets in Positive Supply (Bewley)

The government issues real bonds B, financed by lump-sum taxes:

$$rB = T.$$

The household budget constraint becomes

$$c + a' = Ra + y - T,$$

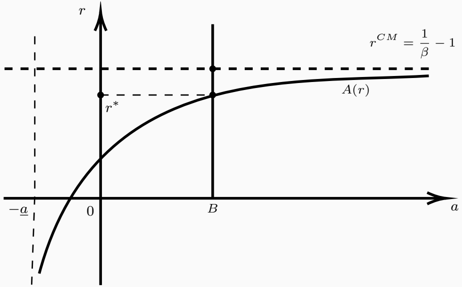
and asset market clearing is

$$\int_{A\times Y} a'(a, y; r)\, d\lambda_r^* = B.$$

Same analysis as before, with one extra endogenous variable (T) and one extra equation. With production, however, public debt also crowds out capital (Aiyagari and McGrattan, 1998).

Bewley: Equilibrium with Positive Supply

Equilibrium of the endowment economy where assets are in fixed positive supply



- The same $A(r)$ curve, now crossing $B > 0$: the resulting r^* is closer to r^{CM}
- As B rises, more liquidity improves self-insurance and the economy moves toward full insurance

The Aiyagari Production Economy

Reinterpret y as efficiency units of labor, supplied inelastically; assets are claims to capital. A representative firm operates CRS technology $F(K, L)$ with depreciation δ. The household budget is $c + a' = Ra + wy$.

Stationary equilibrium: V, policies (a', c), firm choices (K, L), prices (r^*, w^*), and a measure λ^* such that:

1. Given (r, w), the policies a' and c solve the household problem
2. Firm optimality: $r + \delta = F_K(K, L)$ and $w = F_L(K, L)$
3. Labor market clears: $L = \sum_{i=1}^{N} y_i \Pi^*(y_i)$

Aiyagari: Remaining Equilibrium Conditions

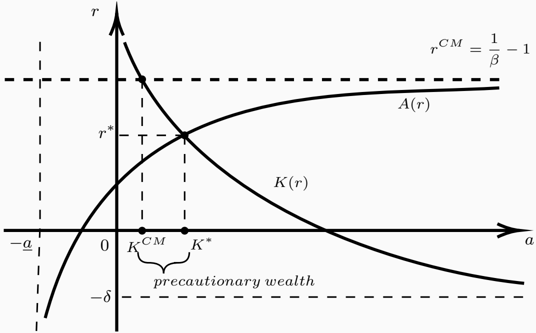
4. Asset market clears: $\int_{A\times Y} a'(a,y)\, d\lambda^* = K$
5. Goods market clears: $\int_{A\times Y} c(a,y)\, d\lambda^* + \delta K = F(K,L)$
6. λ^* is the invariant measure induced by Q

Firm capital demand follows from $F_K(K,L) = r + \delta$: it is continuous and strictly decreasing in r, with $K \to \infty$ as $r \to -\delta$ and $K \to 0$ as $r \to \infty$.

Equilibrium is the intersection of an upward-sloping household asset supply and a downward-sloping firm capital demand.

Equilibrium of the Production Economy

Equilibrium of the production economy



- $A(r)$ crosses $K(r)$ at (K^*, r^*); complete markets give an infinitely elastic supply at $r^{CM} = 1/\beta - 1$, with capital K^{CM}
- $K^* - K^{CM}$ is **precautionary wealth** — extra capital from uninsurable risk

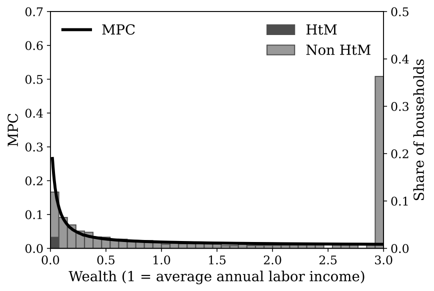
Aiyagari: Quantitative Findings

1. **Self-insurance is effective**: it cuts consumption variability by roughly half relative to autarky
2. **Precautionary saving** adds about 3 percentage points to the aggregate saving rate — and over 10 with higher risk aversion or income volatility
3. **Wealth distribution**: more positively skewed (mean $>$ median) and more dispersed than income, as in the data — but with too much accumulation at the bottom and too little concentration at the very top (Chapter 21)

The third finding is the main quantitative failure of the one-asset model, and motivates the extensions discussed in Chapter 21.

The MPC: Models versus Data

Under the PIH, the quarterly MPC is approximately $r \approx 1\%$, and the MPC out of *anticipated* changes is zero. **Data**: households spend 15–30% of fiscal stimulus payments on nondurables within a quarter (Parker et al., 2013) — up to twenty times the PIH value.



- The MPC declines steeply with wealth; the dark bar near zero is the share of **hand-to-mouth** households

Reconciling Models and Data

In the calibrated one-asset model the average quarterly MPC reaches only about 5%: matching aggregate wealth requires a high β, and optimizing households then stay well away from the constraint.

Two fixes:

- **Discount-factor heterogeneity**: impatient types hold low wealth and have high MPCs
- **Two-asset models** with liquid and illiquid assets, generating *wealthy hand-to-mouth* households — high wealth overall, but little of it liquid (Kaplan and Violante, 2014)

Summary

Key Takeaways (I)

1. **Two benchmarks**: autarky ($c_{i,t} = y_{i,t}$, full pass-through) versus complete markets ($u'(c_i)/u'(c_j) = \lambda_i/\lambda_j$ constant, consumption proportional to aggregate). Tests reject both: **partial insurance**.
2. **Bond economy**: $a_{t+1} = (1+r)(y_t + a_t - c_t)$; self-insurance through saving and dissaving.
3. **Deterministic case**: Euler equation $u'(c_t)/\beta u'(c_{t+1}) = 1 + r$; consumption linear in total wealth; $MPC = 1 - R^{-1}(\beta R)^{1/\sigma} \approx \rho$; MPC out of permanent income $= 1$.

Key Takeaways (II)

4. **PIH** (quadratic utility, $\beta R = 1$): $c_t = \mathbb{E}_t c_{t+1}$, a martingale (Hall, 1978); certainty equivalence; $\Delta c_t = \frac{r}{1+r} u_t + v_t$ — annuity value of transitory shocks, full value of permanent ones.
5. **Blundell-Preston**: ΔVar$(c) \approx$ Var(v) identifies permanent income inequality; **Campbell**: saving for a rainy day.
6. **Borrowing constraints**: Euler inequality $u'(c_t) \geq \beta R \mathbb{E}_t u'(c_{t+1})$; the timing of income matters; constrained $MPC = 1$. The natural limit $-\frac{1+r}{r} y_{min}$ never binds under Inada.

Key Takeaways (III)

7. **Precautionary saving**: two sources — occasionally binding constraints (Figure 11.2) and prudence $u''' > 0$ (Kimball index $-u'''c/u''$; DARA implies prudence). The consumption function is concave under HARA.
8. **Buffer stock** (Carroll): impatience ($\rho > r$) plus risk gives a target wealth x^*, with $\mathbb{E}_t \Delta \log c \approx \frac{1}{\sigma}(r - \rho) + \frac{\sigma+1}{2}\mathsf{Var}_t(\Delta \log c)$.
9. **Bounded wealth**: $\beta R < 1$ with DARA suffices, and holds in equilibrium since $r^* < 1/\beta - 1$.

Key Takeaways (IV)

10. **Huggett, Bewley, Aiyagari**: stationary equilibria in which $A(r)$ crosses zero, B, or $K(r)$ respectively. Precautionary wealth is $K^* - K^{CM}$.
11. **Payoffs of incomplete markets**: a lower equilibrium risk-free rate (resolving the risk-free rate puzzle), effective self-insurance, and a higher aggregate saving rate.
12. **Remaining gap**: model MPCs are still far too low relative to the data, and top wealth concentration too small — motivating discount-factor heterogeneity and two-asset models with wealthy hand-to-mouth households.