Chapter 12: Labor Supply

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Facts About Hours of Work

Motivation

Hours worked are a fundamental input into aggregate production and a crucial determinant of output per capita. Macroeconomists view hours as the equilibrium where labor demand meets labor supply.

Stylized facts to understand: variation in hours **across countries**, **over time** (secular trends and business cycles), **over the life cycle**, and **across demographic groups**.

Three key elements:

1. Identify the incentives and disincentives to work
2. Measure the sensitivity to these incentives: **labor supply elasticities**
3. Aggregate these elasticities to derive macro patterns

Hours per Person Across Countries

Hours of work per person aged 15+ relative to the US, 2015–2019

< 0.75	$(0.75, 0.85)$	$(0.85, 0.95)$	> 0.95
Italy (0.69)	Finland (0.77)	UK (0.85)	Canada (0.96)
France (0.70)	Austria (0.79)	Sweden (0.90)	Australia (0.98)
Belgium (0.72)	Norway (0.80)	Ireland (0.91)	US (1.00)
Greece (0.73)	Netherlands (0.82)	Japan (0.91)	New Zealand (1.07)
Denmark (0.74)	Portugal (0.85)	Switzerland (0.93)	Korea (1.12)
Germany (0.74)			
Spain (0.75)			

US level: 1096 hours per year. Differences of 30% or more between the US and several European countries.

Intensive and Extensive Margins

Labor supply along the intensive and extensive margin, 2015–2019 (selected values)

	Extensive: Emp/Pop (%)	Intensive: Annual hours per employed
Lowest	Greece (40.9), Italy (44.1)	Germany (1388), Denmark (1395)
	Spain (48.6)	Norway (1423), Netherlands (1435)
Middle	US (60.1), UK (60.4)	Spain (1693), Japan (1693)
	Korea (60.7)	Italy (1718)
Highest	Switzerland (65.1)	New Zealand (1761), US (1825)
	New Zealand (66.9), Sweden (67.6)	Greece (1941), Korea (2026)

Reference point: 40 hours/week $\times$ 50 weeks $=$ 2000 annual hours.

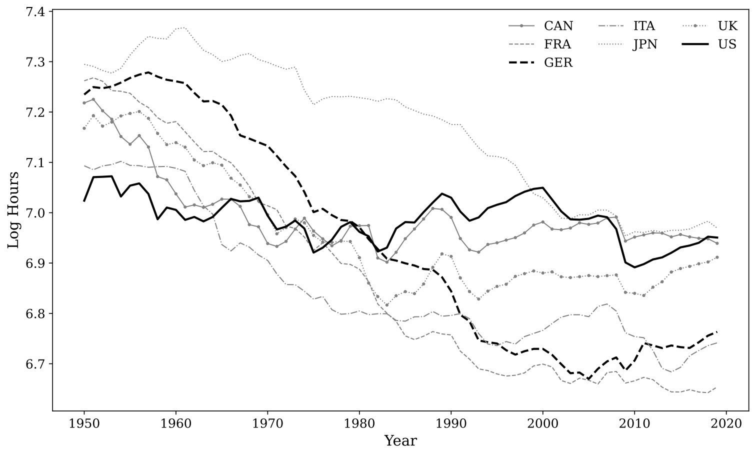
The Two Margins: Reading the Table

Aggregate differences in hours per person combine **both** margins.

- Greece has low employment but very long hours per worker
- Germany and Sweden are the opposite: high employment, short hours
- The US is mid-range on the extensive margin (60.1%) but near the top on the intensive margin (1825 hours)

Hours over Time

Log average annual hours worked per person for the G7 countries, 1950–2019



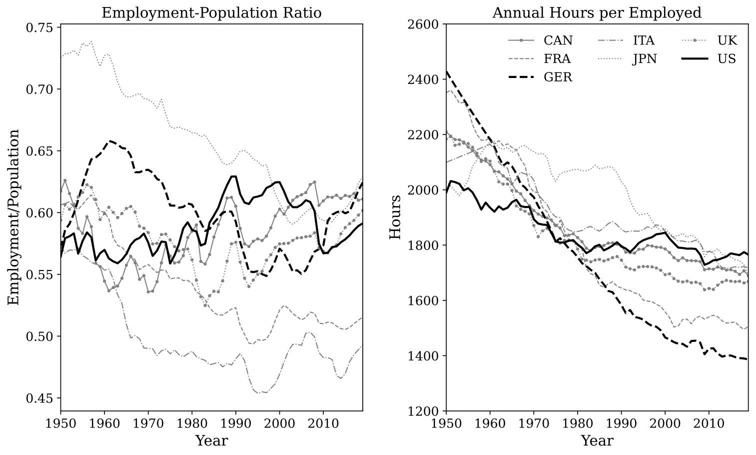
Hours over Time: What the Figure Shows

- Most G7 countries: declines of roughly **30%** over the period
- US: essentially flat, no statistically significant trend — the exception
- In 1950 the US had the **lowest** hours among these countries; by 2019 among the highest

The large cross-country differences in Table 12.1 are therefore recent: they opened up over these seven decades rather than being a long-standing feature of the data.

Hours over Time: The Two Margins

Intensive and extensive margins of labor supply for the G7, 1950–2019



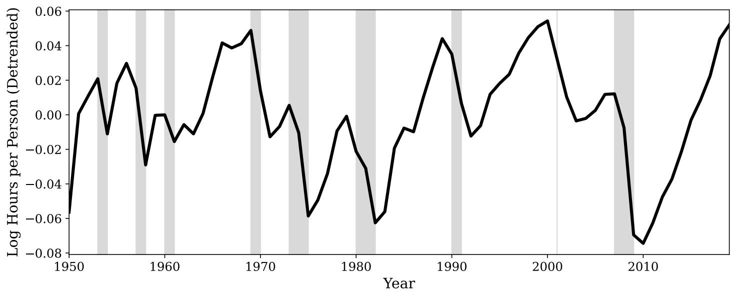
The Two Margins: What the Figure Shows

Left panel: employment to population ratio. Right panel: annual hours per employed person.

- Intensive margin: all countries decline, though the magnitude varies widely (Germany $\approx 40\%$, US $\approx 10\%$)
- Extensive margin: much more varied — some countries decline significantly, some show little secular change, and others increase

Business Cycle Fluctuations

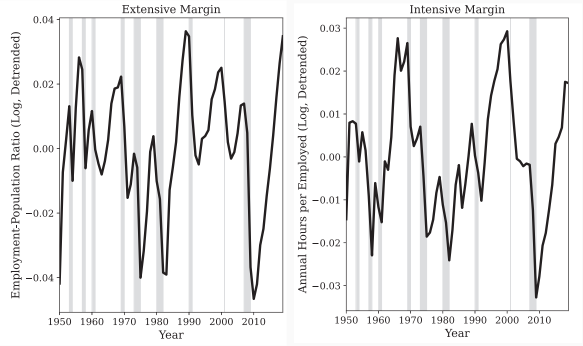
Business cycle fluctuations in hours worked per person, US, 1950–2019



- Residual of a regression of log hours on a quartic time polynomial; NBER recessions shaded
- Peak-to-trough movements exceed ten percentage points in many episodes

Business Cycles: The Two Margins

Extensive (left) and intensive (right) margins, US, 1950–2019



- Each panel detrended on a quartic polynomial in time. Both margins fluctuate substantially; the **extensive margin movements are somewhat larger**

Hours Across Demographic Groups

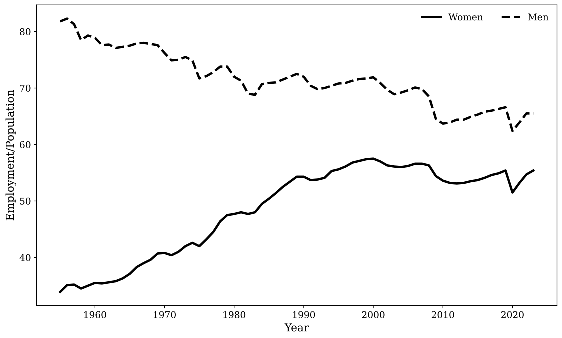
Hours of work per day, by age and gender (ATUS 2023)

Age	All	Male	Female
15–19	1.27	1.24	1.29
20–24	3.67	3.81	3.52
25–34	5.02	5.90	4.13
35–44	5.04	5.79	4.29
45–54	4.79	5.45	4.15
55–64	4.02	4.73	3.35
65–74	1.42	1.88	1.02
75+	0.37	0.55	0.23
Total	3.56	4.17	2.98

Pronounced **hump shape** over the life cycle for both genders, peaking at ages 25–54.

Gender: Employment to Population

Employment to population for the US by gender



- Male employment/population: down more than 10 percentage points
- Female: up roughly 20 percentage points — a pervasive feature of the data

Education

Hours of work per day by education (ATUS 2023)

$<$HS	HS	SC	C	$>$C
2.89	3.39	3.90	4.08	4.17

Work time rises substantially with education: college-educated workers devote roughly 20% more time to work than high-school-educated workers.

Static Models of Labor Supply

Benchmark Static Model

Separable preferences:

$$U(c,\ell) = u(c) - v(\ell)$$
$$u(c) = \frac{1}{1-\sigma}c^{1-\sigma}, \qquad v(\ell) = \frac{\psi}{1+1/\gamma}\ell^{1+1/\gamma}$$

Budget: $c = w\ell + I$, where I is non-labor income. First-order condition:

$$\frac{\psi \ell^{1/\gamma}}{(w\ell + I)^{-\sigma}} = w$$

MRS between consumption and time = the real wage (the price of time).

Income and Marshallian Elasticities

Define elasticities of the solution $\ell(w, I)$:

$$\varepsilon^M_{\ell,w} = \frac{w}{\ell}\ell_1(w, I), \qquad \varepsilon_{\ell,I} = \frac{I}{\ell}\ell_2(w, I)$$

With $s_I = I/(w\ell + I)$ the non-labor income share:

Income elasticity:

$$\varepsilon_{\ell,I} = -\frac{\sigma s_I}{1/\gamma + \sigma(1 - s_I)} \leq 0$$

Higher non-labor income $\Rightarrow$ higher consumption $\Rightarrow$ lower marginal benefit of working.

Marshallian Elasticity

Marshallian (uncompensated) elasticity:

$$\varepsilon^M_{\ell,w} = \frac{1 - \sigma(1 - s_I)}{1/\gamma + \sigma(1 - s_I)}$$

Sign ambiguous: **substitution effect** (higher reward to work) vs. **income effect** (higher consumption at given hours).

Special case $I = 0$: $\varepsilon^M_{\ell,w} = (1 - \sigma)/(1/\gamma + \sigma)$. Positive iff $\sigma < 1$; exactly zero at $\sigma = 1$ (effects offset, each with magnitude increasing in γ).

Hicksian Elasticity and the Slutsky Equation

The **Hicksian (compensated) elasticity** holds utility constant — pure substitution effect. From the Slutsky decomposition:

$$\varepsilon^H_{\ell,w} = \varepsilon^M_{\ell,w} - \frac{1 - s_I}{s_I}\varepsilon_{\ell,I}$$

Substituting the expressions:

$$\varepsilon^H_{\ell,w} = \frac{1}{1/\gamma + \sigma(1 - s_I)} > 0$$

Always positive, and always weakly larger than the Marshallian.

Elasticities and Tax Policy

Mapping to tax policy ($c = (1-\tau)w\ell + y + T$):

- Change in τ alone = change in w $\Rightarrow$ **Marshallian**
- Change in transfer T alone = change in I $\Rightarrow$ **income elasticity**
- Budget-neutral reform ($T = \tau w\ell$): income effect washes out $\Rightarrow$ **Hicksian** (Prescott, 2004)

Extension 1: Home Production

Two uses of work time: market ℓ_m (wage w) and home ℓ_h (productivity A, $y_h = A\ell_h$). Composite consumption:

$$c = g(c_m, c_h) = \left[\theta c_m^\eta + (1-\theta)c_h^\eta\right]^{1/\eta}, \qquad 0 < \eta < 1$$

FOCs imply MRS = MRT: $g_2/g_1 = w/A$. With $I = 0$:

$$\frac{\ell_h}{\ell_m} = \left(\frac{1-\theta}{\theta}\right)^{\frac{1}{\eta-1}} \left[\frac{w}{A}\right]^{\frac{\eta}{\eta-1}}$$

Since $\eta \in (0,1)$, the exponent on w/A is negative: home work falls relative to market work as the market wage rises relative to home productivity.

Extension 1: Home Production (cont.)

Greenwood, Seshadri, and Yorukoglu (2005): technological change in home production (e.g., appliances) was a major driver of rising female labor force participation.

Extension 2: Household Labor Supply

Unitary household: members f and m share one objective:

$$U(c, \ell_f, \ell_m) = u(c) - v(\ell_f) - v(\ell_m), \qquad c = w_f \ell_f + w_m \ell_m + I$$

FOCs: $u'(c) w_j = v'(\ell_j)$, $j = f, m$. Each member's labor supply depends on the **other member's wage** (acts like non-labor income). Dividing the FOCs:

$$\frac{\ell_f}{\ell_m} = \left(\frac{w_f}{w_m}\right)^{\gamma}$$

Extension 2: Household Labor Supply (cont.)

- γ also governs **within-household reallocation**: a force toward specialization
- A falling gender wage gap raises married female labor supply relative to male
- **Added worker effect**: a fall in ℓ_m (e.g., unemployment) raises ℓ_f

Extension 3: Productivity of Leisure

Define leisure $l = \bar{\ell} - \ell$ and $\tilde{U}(c, l) = u(c) + \tilde{v}(l)$ with $\tilde{v}(l) = \frac{1}{1-1/\gamma} l^{1-1/\gamma}$.

- Corner solutions now possible: with high non-labor income, $\ell = 0$ can be optimal
- Elasticity of utility with respect to work is no longer constant

Extension 3: Leisure Activities

Leisure activities (Aguiar, Bils, Hurst, and Charles, 2021): J activities with productivities A_j:

$$\tilde{v}\left(\sum_{j=1}^{J} \frac{(A_j l_j)^{1-\delta}}{1-\delta}\right)$$

A rise in some A_j raises the value of leisure. Key finding: a large increase in the productivity of leisure time in computer/video games led to a large increase in leisure among young males.

Dynamic Labor Supply: A First Look

Two periods, $\beta = 1/(1+r)$, lifetime budget:

$$c_1 + \frac{c_2}{1+r} = w_1\ell_1 + \frac{w_2\ell_2}{1+r} + I$$

FOCs with multiplier λ: $c_t^{-\sigma} = \lambda$ and $\psi\ell_t^{1/\gamma} = \lambda w_t$. Dividing:

$$\log \ell_1 - \log \ell_2 = \gamma \left[\log w_1 - \log w_2\right]$$

Work relatively more when wages are relatively high. The strength of this **intertemporal substitution** is governed solely by γ: the **Frisch elasticity** (holding the marginal utility of wealth λ fixed).

This equation underlies panel-data estimation of γ and is central to business cycle analysis.

Dynamic Models: Theory and Estimation

The Life-Cycle Model and Two-Stage Budgeting

$$\max_{\{c_{it},\ell_{it}\}_{t=0}^{T}} \mathbb{E}_0 \sum_{t=0}^{T} \beta^t U(c_{it}, \ell_{it})$$

subject to

$$c_{it} + a_{i,t+1} = Ra_{it} + (1-\tau)w_{it}\ell_{it} + T, \qquad a_{i,t+1} \geq -\underline{a}_i$$

Two-stage budgeting: (1) choose savings via the Euler equation; (2) given $a_{i,t+1}$, define net non-labor income

$$I_{it} = ra_{it} + T - (a_{i,t+1} - a_{it})$$

so the period budget becomes $c_{it} = (1-\tau)w_{it}\ell_{it} + I_{it}$ — isomorphic to the static model.

Two-Stage Budgeting: The Result

Result: the Marshallian and Hicksian elasticities take the *same* expressions as in the static model:

$$\varepsilon^M_{\ell,w} = \frac{1-\sigma(1-s_{It})}{1/\gamma+\sigma(1-s_{It})}, \qquad \varepsilon^H_{\ell,w} = \frac{1}{1/\gamma+\sigma(1-s_{It})}$$

These are fundamentally **static** concepts, relevant for **permanent** wage or tax changes.

The Frisch Elasticity

Response of hours to a wage change **holding the marginal utility of wealth λ_{it} constant**: the relevant concept for **transitory or anticipated** changes (negligible wealth effects, no news).

General formula from the FOCs $U_c = \lambda_{it}$ and $-U_\ell = \lambda_{it}(1-\tau)w_{it}$:

$$\varepsilon^F_{\ell,w} = \frac{U_\ell}{\ell_{it}U_{\ell\ell} - \ell_{it}\left(U^2_{\ell c}/U_{cc}\right)}$$

The Frisch Elasticity: Functional Forms

Separable utility $U = \frac{c^{1-\sigma}-1}{1-\sigma} - \psi \frac{\ell^{1+1/\gamma}}{1+1/\gamma}$: $\varepsilon^F_{\ell,w} = \gamma$. The two curvature parameters control the two key elasticities ($1/\sigma$ and γ). Ordering:

$$\varepsilon^F_{\ell,w} \geq \varepsilon^H_{\ell,w} \geq \varepsilon^M_{\ell,w}$$

GHH utility $U = \frac{1}{1-\sigma}\left(c - \psi \frac{\ell^{1+1/\gamma}}{1+1/\gamma}\right)^{1-\sigma}$: zero income effect, so $\varepsilon^M = \varepsilon^H = \varepsilon^F = \gamma$.

Non-Constant Frisch Elasticities

Not all utility functions imply a constant Frisch elasticity. For

$$U = \frac{\left(c^{1-\psi}(1-\ell)^{\psi}\right)^{1-\sigma} - 1}{1-\sigma}:$$

$$\varepsilon^F_{\ell,w} = \left(\frac{1-\ell}{\ell}\right)\frac{1-(1-\psi)(1-\sigma)}{\sigma}$$

Non-Constant Frisch Elasticities (cont.)

For

$$U = \frac{c^{1-\sigma} - 1}{1-\sigma} + \psi \frac{(1-\ell)^{1-\gamma}}{1-\gamma}:$$

$$\varepsilon^F_{\ell,w} = \left(\frac{1-\ell}{\ell}\right)\frac{1}{\gamma}$$

In both cases the Frisch elasticity of *leisure* is constant, and individuals who work longer hours are **less responsive** to temporary wage changes.

Frisch Elasticity and Wage Growth

With separable utility and no constraints, the Euler equation $c_{it}^{-\sigma} = \beta R c_{i,t+1}^{-\sigma}$ combined with the intratemporal FOC gives

$$\frac{d\log(\ell_{i,t+1}/\ell_{it})}{d\log(w_{i,t+1}/w_{it})} = \gamma$$

Structural Estimation of γ: The λ Problem

Taking logs of the FOCs:

$$\log c_{it} = -\frac{1}{\sigma} \log \lambda_{it}, \qquad \log \ell_{it} = const_i + \gamma \log w_{it} + \gamma \log \lambda_{it}$$

λ_{it} is **unobservable** and correlated with wages (persistent wage shocks raise wealth) $\Rightarrow$ OLS is **downward biased**. Three responses:

1. Instrumental variables (MaCurdy, 1981): first-difference and instrument $\Delta \log w_{it}$ with lagged wage growth. Weak in practice: division bias from measurement error in hours; near-zero autocorrelation of wage growth when wages are persistent.

Structural Estimation: Two More Responses

2. Wage growth expectations (Pistaferri, 2003):
$\Delta \log \ell_{it} = \gamma \mathbb{E}_{t-1}[\Delta \log w_{it}] + \gamma(\varepsilon_{it} + \Delta \log \lambda_{it})$. Under PIH, $\Delta \log \lambda_{it}$ responds only to news. Italian subjective expectations data: $\gamma = 0.7$.

3. Consumption data (Altonji, 1986):
$\Delta \log \ell_{it} = \gamma \Delta \log w_{it} - \gamma\sigma \Delta \log c_{it}$. Robust to constraints (envelope condition), but consumption is measured with error.

Biases from Constraints and Precaution

Domeij and Floden (2006): binding borrowing constraints break both IV and expectations approaches.

- Steep predicted wage growth $\Rightarrow$ more likely constrained $\Rightarrow$ negative correlation between $\mathbb{E}_{t-1}[\Delta \log w]$ and $\Delta \log \lambda$
- Constrained agents hit by negative transitory shocks *increase* labor supply as wages fall

Both forces bias $\hat{\gamma}$ **downward**.

Precautionary Labor Supply and Experiments

Low (2005): even without binding constraints, uninsurable risk generates **precautionary labor supply** — working more to build savings — another downward omitted-variable bias if risk is high when wage growth is high (e.g., the young).

Experimental evidence: Fehr and Goette (2007), bike messengers in Zurich, transitory 25% commission increase: elasticity > 1. Bianchi, Gudmundsson, and Zoega (2001), Iceland's 1987 tax-free year: $\gamma \approx 0.4$.

Optimization Frictions (Chetty, 2012)

Static GHH problem with a fixed utility cost Δ of changing hours:

$$\max_{c,\ell} \log c - \psi \frac{\ell^{1+1/\gamma}}{1+1/\gamma} - \Delta \mathbb{I}_{\{\ell \neq \ell^*\}} \quad \text{s.t.} \quad c = w\ell$$

Optimal hours: $\ell^* = (w/\psi)^\gamma$. After a wage change of size ε, the utility gain from re-optimizing hours (vs. only adjusting consumption) is **second order**:

$$U(c^*_\varepsilon, \ell^*_\varepsilon) - U(c^*_\varepsilon, \ell^*) \approx \frac{1}{2}\gamma(1+\gamma)\varepsilon^2$$

Optimization Frictions: Implication

Hours adjust only if

$$\varepsilon > \varepsilon_{min} = \left[\frac{2\Delta}{\gamma(1+\gamma)}\right]^{1/2}$$

$\Rightarrow$ Hours don't respond to **small** wage changes even when the true γ is large: downward bias in micro estimates. Identification requires **large** wage variation.

Balanced Growth

Balanced Growth Restrictions on Preferences

On a balanced growth path, C, I, K, Y all grow at rate g, but hours **cannot** grow forever (bounded by the time endowment).

King, Plosser, and Rebelo (1988): the period utility $U(C, 1-L)$ is consistent with constant hours along a BGP if and only if

$$U(C, 1-L) = \begin{cases} \dfrac{C^{1-\sigma}-1}{1-\sigma} v(1-L) & \text{if } \sigma \neq 1 \\ \log(C) + v(1-L) & \text{if } \sigma = 1 \end{cases}$$

Balanced Growth: Implications

These **balanced-growth (KPR) preferences** imply that income and substitution effects of a permanent wage increase exactly cancel.

But Figure 12.1 shows hours *declining* in most countries: the flat US profile is the exception rather than the rule. Alternatives are discussed below (Boppart-Krusell).

Labor Supply Across Countries and over the Cycle

Prescott (2004): Taxes and Cross-Country Hours

Representative household with $U = \log C_t - \frac{\psi}{1+1/\gamma} L_t^{1+1/\gamma}$, taxes τ_c on consumption and τ_ℓ on labor, lump-sum rebate; Cobb-Douglas production.

Hours along the BGP:

$$L^* = \left[\frac{(1-\tau)(1-\alpha)}{\chi\psi}\right]^{\frac{\gamma}{1+\gamma}}, \qquad \tau = \frac{\tau_\ell + \tau_c}{1+\tau_c}, \qquad \chi = \frac{C}{Y}$$

Only τ and χ vary across countries (with common α, ψ, γ).

Prescott (2004): Quantitative Fit

US vs. Italy (mid-1990s): hours ratio in the data $= 1.57$. With τ: 0.40 (US), 0.64 (Italy); χ: 0.81 (US), 0.69 (Italy); $\gamma = 1$: predicted ratio $= 1.42$.

Taxes account for much of the cross-country variation in hours. The relevant elasticity for these permanent, rebated changes is the **Hicksian**.

Income Effects and Declining-Hours Growth Paths

Bick, Fuchs-Schündeln, and Lagakos (2018): hours are systematically **higher in poorer countries** — suggesting income effects exceed substitution effects. Two modifications:

Subsistence consumption:

$$U = \log(C_t - \bar{C}) - \frac{\psi}{1 + 1/\gamma} L_t^{1+1/\gamma}$$

At low wages, individuals work long hours to reach $\bar{C}$; the excess income effect vanishes as C grows.

Boppart-Krusell: Declining-Hours BGP

Boppart and Krusell (2020):

$$U = \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{\psi}{1+1/\gamma} L_t^{1+1/\gamma}, \qquad \sigma > 1$$

Income effect always dominates: a BGP with hours **declining at a constant rate** — a better fit to the long-run data.

Taxes vs. Income Effects: An Open Question

Ohanian, Raffo, and Rogerson (2008) offer an alternative interpretation of the trend decline in hours: **rising tax rates** can account for much of the post-1960 trend behavior of hours among OECD economies.

Whether income effects dominate substitution effects — and if so, whether the excess diminishes as consumption grows — remains an open issue.

Labor Supply over the Business Cycle

With stochastic Z_t, the hours condition becomes (in logs):

$$\log L_t = const - \frac{\gamma}{1+\gamma} \log \left(\frac{C_t}{Y_t} \right)$$

In a boom with mean-reverting productivity: consumption rises less than output (PIH) $\Rightarrow$ C_t/Y_t falls $\Rightarrow$ hours rise. The strength is governed by $\gamma/(1+\gamma)$:

- $\gamma \to \infty$ (indivisible labor): one-for-one response
- $\gamma = 1$: already **half** the infinite-elasticity response

Business Cycle Hours: Capital and the Labor Wedge

Without capital, $C_t = Y_t$ and hours never move: capital/saving is essential for procyclical hours under KPR preferences.

The labor wedge: empirically, hours fall in recessions by more than C/Y movements predict. The missing term acts like a countercyclical labor income tax — pointing to labor market frictions (Chapters 14 and 20).

Human Capital and Dynamic Returns

Imai-Keane: Learning by Doing

Add dynamic returns to work: the wage depends on accumulated experience:

$$w_{it} = \left(1 + \kappa \sum_{j=0}^{t-1} \ell_{ij}\right) w_{i0}$$

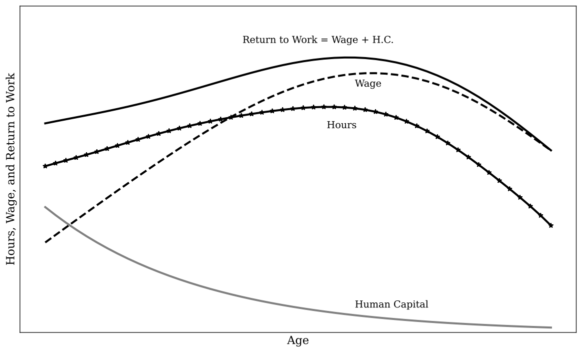
FOC for hours at t:

$$\psi \ell_{it}^{1/\gamma} = \lambda_{it}(1-\tau) \left[w_{it} + \kappa w_{i0} \sum_{j=1}^{T-t} R^{-j} \mathbb{E}_t[\ell_{i,t+j}] \right]$$

The **human capital term** (second term) is a wedge between the current wage and the **total return to work**. The parameter γ is the elasticity with respect to the total return, not the wage.

Life-Cycle Implications

Life-cycle profile of hours, wages, the human capital term, and the return to work



Life-Cycle Implications: Interpretation

- The human capital term is large when young (more years to enjoy higher wages) and declines with age
- Hours are flat while wages grow steeply early in life: the **return to work** is much flatter than the wage
- Naive estimation from wages and hours **underestimates** γ
- The effective Frisch elasticity **rises with age**: Imai-Keane estimate 0.4 (young) to 2.0 (old)

Human capital can also **flip the elasticity ordering**: a permanent tax cut raises both the wage and the human capital term, strengthening the substitution effect; it dampens responses to transitory changes but magnifies responses to permanent ones.

Intensive and Extensive Margins

Indivisible Labor: The Static Model

Unit mass of identical agents, $Y = AL$, separable preferences.
Indivisible labor: $\ell \in \{0, \bar{\ell}\}$.

The planner chooses the employed fraction e and common consumption c (lotteries convexify):

$$\max_{e,c}\ eU(c,\bar{\ell}) + (1-e)U(c,0) \quad \text{s.t.} \quad c = A\bar{\ell}e$$

With separability, the objective reduces to:

$$u(c) - \tilde{\psi}e, \qquad \tilde{\psi} \equiv v(\bar{\ell}) - v(0)$$

Indivisible Labor: The Infinite Aggregate Elasticity

Disutility is **linear in** $e \Rightarrow$ the **aggregate Frisch elasticity is infinite** (Hansen, 1985; Rogerson, 1988), regardless of individual preferences.

Two takeaways: (i) aggregate and individual labor supply are disconnected; (ii) the aggregate Frisch elasticity can be very large independently of the individual one.

Decentralization

Employment lotteries (Hansen-Rogerson): individuals sell employment probabilities; earnings $e\bar{\ell}w$; by the law of large numbers aggregate labor supply is deterministic.

Decentralization: Time Averaging

Time averaging (Ljungqvist and Sargent, 2006): continuous time over $[0, T]$, no discounting. Choosing to work with probability e at each instant is equivalent to **deterministically working a fraction e of one's life**:

$$\max \int_0^T U(c(t), e(t)\bar{\ell})\, dt$$

$$\text{s.t.} \quad \int_0^T c(t)\, dt = \int_0^T e(t)\bar{\ell} A\, dt, \qquad e(t) \in \{0, 1\}$$

With borrowing and saving at the (equilibrium) zero interest rate, individuals smooth consumption at $e^*\bar{\ell}w$ and are indifferent about *when* they work. No lottery markets needed.

Heterogeneity: From Infinite to Finite Aggregate Elasticity

Let disutility ψ be drawn i.i.d. from cdf F. The planner employs those with the lowest draws: reservation level $\psi^*(e)$ with $F(\psi^*) = e$. Total disutility:

$$\tilde{v}(e) = \int_0^{\psi^*(e)} \psi f(\psi) d\psi$$

$\tilde{v}$ is **no longer linear**: the aggregate Frisch elasticity depends on the local properties of the density f — i.e., on the **distribution of reservation wages**.

Chang and Kim (2006)

Chang and Kim (2006): calibrated heterogeneous-agent model with indivisible labor, idiosyncratic productivity ($\log z_{jt+1} = \rho_j \log z_{jt} + \varepsilon_{jt+1}$), and incomplete markets. Panel regression $\log \ell_{it} = \gamma(\log w_{it} - \log c_{it}) + \varepsilon_{it}$ on model data:

- Micro estimates: $\gamma = 0.41$ (men), 0.78 (women)
- Same regression on aggregate time series: $\gamma = 1.08$
- Representative-agent equivalent: $\tilde{\gamma} \approx 2$ — five times the male micro estimate

Both Margins: Nonconvex Mappings

Two specifications mapping hours to efficiency units:

$$\ell_e = \max(0, \ell - \ell_f) \quad \text{(fixed time cost)}$$
$$\ell_e = \ell^\theta,\ \theta > 1 \quad \text{(convex mapping)}$$

Both incentivize **concentrating** hours (avoiding short workweeks) — capturing the essence of indivisibility, with $\bar{\ell}$ endogenous.

Rogerson and Wallenius (2009): life-cycle model with fixed cost, deterministic hump-shaped productivity $z(t)$. Threshold z^*: work only when $z(t) > z^*$, with hours increasing in productivity — **both margins** operate over the life cycle.

Both Margins: Quantitative Results

Rogerson-Wallenius quantitative findings ($u = \log c$, $v = \psi\ell^{1+1/\gamma}/(1+1/\gamma)$, γ ranging over $[0.1, 2]$, recalibrating to life-cycle facts each time):

- Raising the tax rate from 30% to 50% lowers aggregate hours by ∼20% — **roughly independent of** $\gamma \in [0.1, 2]$
- γ controls the intensive/extensive *split*: at $\gamma = 2$ the intensive margin is over 60% of the response; at $\gamma = 0.1$ under 5%

Both Margins: Implications for Estimation

A researcher using the intensive-only benchmark model on aggregate data from two such economies would infer a γ **more than an order of magnitude larger** than the true value.

Erosa, Fuster, and Kambourov (2016): rich version matching male wage/hours data (multiple heterogeneity sources, multiple nonconvexities, time aggregation, measurement error); aggregate elasticity to a temporary unanticipated wage change $= 1.75$.

Summary

Key Takeaways (I)

1. **Facts**: Hours per person vary by 30%+ across countries (Tables 12.1–12.2, both margins); most countries trend down while the US is flat (Figure 12.1); large procyclical fluctuations, more on the extensive margin; hump-shaped life-cycle profile; hours rise with education.
2. **Three elasticities**: Marshallian $\varepsilon^M = \frac{1-\sigma(1-s_I)}{1/\gamma+\sigma(1-s_I)}$ (uncompensated; permanent unrebated changes); Hicksian $\varepsilon^H = \frac{1}{1/\gamma+\sigma(1-s_I)} > 0$ (compensated; budget-neutral reforms); Frisch $\varepsilon^F = \gamma$ for separable utility (constant-λ; transitory/anticipated changes). Ordering: $\varepsilon^F \geq \varepsilon^H \geq \varepsilon^M$. GHH: all equal γ.

Key Takeaways (II)

3. **Extensions**: home production (ℓ_h/ℓ_m falls in w/A); unitary household ($\ell_f/\ell_m = (w_f/w_m)^\gamma$, specialization, added worker effect); leisure productivity (video games).
4. **Estimation**: unobserved λ biases OLS down; IV (weak), subjective expectations ($\gamma = 0.7$), consumption data; borrowing constraints and precautionary labor supply bias down further; Chetty frictions: hours respond only if $\varepsilon > [2\Delta/\gamma(1+\gamma)]^{1/2}$.

Key Takeaways (III)

5. **Balanced growth**: KPR preferences ($C^{1-\sigma}v(1-L)/(1-\sigma)$ or $\log C + v(1-L)$) are necessary and sufficient for constant hours; Boppart-Krusell ($\sigma > 1$) delivers declining-hours BGPs.
6. **Prescott taxes**: $L^* = [(1-\tau)(1-\alpha)/(\chi\psi)]^{\gamma/(1+\gamma)}$; tax differences explain much of US–Europe hours gaps (US/Italy: 1.42 predicted vs. 1.57 data).

Key Takeaways (IV)

7. **Business cycles**: $\log L_t = const - \frac{\gamma}{1+\gamma}\log(C_t/Y_t)$; capital is essential; the residual labor wedge points to frictions.
8. **Human capital** (Imai-Keane): return to work = wage + human capital term; naive estimation underestimates γ; Frisch elasticity rises with age (0.4 to 2.0); elasticity ordering can flip.
9. **Extensive margin**: indivisible labor + lotteries (or time averaging) $\Rightarrow$ infinite aggregate Frisch; heterogeneity makes it finite, governed by the reservation-wage distribution; Chang-Kim: micro 0.41/0.78 vs. aggregate ≈ 2; Rogerson-Wallenius: aggregate Hicksian response roughly independent of γ; micro panel estimates are not informative about aggregate elasticities.