Chapter 17: Money

Chapter authors: Andreas Hornstein and Per Krusell

August 2026

Outline

Introduction

Motivation

Why does an intrinsically useless piece of paper have value? How do markets and government policy jointly determine its value?

- Fiat money: no intrinsic value, not backed by gold
- People accept it because they expect others to accept it
- Money is an **asset without a dividend**—its value is a bubble

Motivation: Key Themes

Key themes of the chapter:

- **Equilibrium indeterminacy**: multiple equilibria, including hyperinflation
- **Wallace's dictum**: in a theory *of* money, the use of money should be an outcome, not an assumption
- Theories *of* money (OLG, missing assets, search) vs. theories *with* money (CIA, MIU, transactions costs)

Three Roles of Money

1. **Store of value**: Transfer resources across time (OLG, missing assets)
2. **Medium of exchange**: Facilitate transactions (CIA, Kiyotaki-Wright)
3. **Unit of account**: Prices quoted in nominal terms (New Keynesian, Chapter 18)

This chapter assumes **flexible prices** throughout. Sticky prices (and hence the unit-of-account role) are deferred to Chapter 18.

The New Keynesian model is cashless: money is absent but prices are nominal. The **cashless limit** discussed here motivates this approach.

Money in Overlapping Generations

OLG Endowment Economy: Setup

Two-period lives, endowments (ω_y, ω_o), log utility: $\log c_y + \log c_o$. Initial old own M units of fiat money. Let $p_{m,t}$ = price of money in terms of goods.

Young agent's problem:

$$\max_{c_y, c_o, M'} \log c_y + \log c_o$$

subject to

$$c_y + p_{m,t} M' = \omega_y, \quad c_o = \omega_o + p_{m,t+1} M', \quad M' \geq 0$$

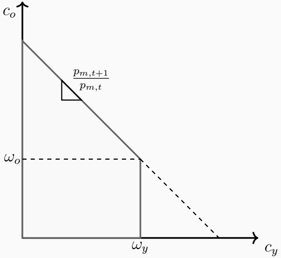
OLG: The Budget Constraint

Combining constraints (if $p_{m,t} > 0$): Writing $R_{m,t} \equiv p_{m,t+1}/p_{m,t}$ for the gross real return on money,

$$c_y + \frac{c_o}{R_{m,t}} = \omega_y + \frac{\omega_o}{R_{m,t}}, \qquad c_y \leq \omega_y.$$

Budget Set and Money Demand

Budget set in the economy with fiat money



Kink at $(c_y, c_o) = (\omega_y, \omega_o)$ because $M' \geq 0$.

Individual Money Demand

Individual money demand:

$$p_{m,t}M_t' = \max\left\{\frac{1}{2}\left(\omega_y - \frac{\omega_o}{p_{m,t+1}/p_{m,t}}\right), 0\right\}$$

Market clearing: $M_t' = M$ for all t.

Equilibrium Price Dynamics

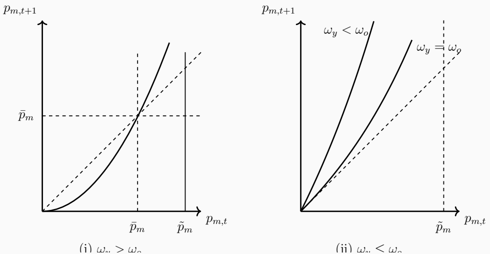
Combining demand and market clearing when $p_{m,t} > 0$:

$$p_{m,t+1} = \frac{\omega_o \, p_{m,t}}{\omega_y - 2M \, p_{m,t}}$$

Stationary monetary equilibrium: $\bar{p}_m = (\omega_y - \omega_o)/(2M)$, positive only if $\omega_y > \omega_o$.

Price Dynamics

Panel (i): $\omega_y > \omega_o$. Panel (ii): $\omega_y \leq \omega_o$.



- Case (i): two steady states, $p_{m,t} = 0$ and $p_{m,t} = \bar{p}_m$; a continuum of equilibria indexed by $p_{m,0} \in [0, \bar{p}_m]$
- Case (ii): only the origin, so money never has value

Multiple Equilibria and Indeterminacy

Case $\omega_y > \omega_o$: Continuum of equilibria indexed by $p_{m,0} \in [0, \bar{p}_m]$.

- $p_{m,0} = \bar{p}_m$: stationary monetary equilibrium (money retains value)
- $p_{m,0} \in (0, \bar{p}_m)$: money steadily loses value, converges to zero
- $p_{m,0} = 0$: non-monetary equilibrium (money never has value)
- $p_{m,0} > \bar{p}_m$: no equilibrium (prices explode)

Case $\omega_y \leq \omega_o$: Only the non-monetary equilibrium exists ($\bar{p}_m \leq 0$).

The number of nominal units M has no real importance: only $m_t \equiv p_{m,t} M$ matters.

Welfare Across Equilibria

Stationary monetary equilibrium: Pareto efficient (gross return on money $= 1$, satisfies Balasko-Shell condition from Chapter 6).

All non-stationary monetary equilibria: Pareto inefficient (return < 1 asymptotically), but still Pareto-improve on autarky. Ranked: higher $p_{m,0} \Rightarrow$ higher utility for all generations.

Welfare: Capital and Money Growth

With capital: Money can coexist with physical capital if the autarky interest rate is below zero. In such cases money and capital earn the same return asymptotically.

With money growth at gross rate $1 + \mu$ (lump-sum transfers): Stationary equilibrium has real return $1/(1 + \mu)$ on money. The real value of total money stock is lower.

Money in Dynastic Models

Fiat Money Has No Value

Household budget with money, bonds, and a real asset:

$$c_t + q_t a_{t+1} + p_{m,t} M_{t+1} + p_{m,t} B_{t+1}$$
$$= w_t(1 - l_t) + a_t + p_{m,t} M_t + p_{m,t}(1 + i_{t-1}) B_t - \tau_t$$

No-arbitrage between assets requires:

$$q_t = \frac{p_{m,t}}{p_{m,t+1}} = \frac{p_{m,t}}{p_{m,t+1}} \frac{1}{1 + i_t}$$

If $i_t > 0$: money is dominated in return by bonds $\Rightarrow$ no one holds money $\Rightarrow$ $p_{m,t} = 0$.

Fiat Money Has No Value: The TVC Argument

If $i_t = 0$ for all t: money and bonds equivalent. TVC:

$$0 \geq \lim_{T\to\infty} q_0 q_1 \cdots q_T \, m_T = p_{m,0} \lim_{T\to\infty} M_T$$

If M_T does not vanish: money cannot have value. In dynastic models, fiat money has no value as a store of value.

Cash-in-Advance Model

Give money value by assumption: goods can only be purchased with money.

CIA constraint: $c_t \leq m_t$ (where $m_t = p_{m,t} M_t$)

Budget: $c_t + q_t a_{t+1} + \frac{p_{m,t}}{p_{m,t+1}} m_{t+1} = a_t + m_t + w_t(1 - l_t) - \tau_t$

FOCs:

$$u_1(c_t, l_t) = \lambda_t + \mu_t, \qquad u_2(c_t, l_t) = \lambda_t w_t$$

$$\lambda_t q_t = \beta \lambda_{t+1}, \qquad \lambda_t \frac{p_{m,t}}{p_{m,t+1}} = \beta(\lambda_{t+1} + \mu_{t+1})$$

where μ_t is the multiplier on the CIA constraint. The CIA must bind in any stationary equilibrium (otherwise TVC is violated).

CIA: Steady State and Distortions

In stationary equilibrium with constant M and z: $q = \beta$, $p_m = c/M$ constant, no inflation, $i = 1/\beta - 1 > 0$.

Combining consumption and labor FOCs:

$$\frac{u_2(c, 1 - c/z)}{u_1(c, 1 - c/z)} = \frac{\lambda}{\lambda + \mu} z = \beta z$$

The MRS between leisure and consumption is $\beta z < z$ (the MRT). The CIA acts like a tax on labor: today's earnings can only be used tomorrow.

$\Rightarrow$ Leisure too high, consumption too low relative to the frictionless economy.

The Friedman Rule

Shrink the money stock at rate $\gamma = \beta$ (financed by lump-sum taxes). Then:

- Real return on money $= 1/\gamma = 1/\beta =$ real interest rate
- Nominal interest rate $= 0$
- Opportunity cost of holding money $= 0$
- CIA no longer distorts $\Rightarrow$ first-best allocation
- TVC satisfied because $M_T \to 0$

Friedman rule: Optimal monetary policy equalizes the return on money and bonds by deflating at the rate of time preference. Equivalently: "pay interest on money."

Money in the Utility Function and Transactions Costs

Money in utility (MIU): Period utility $u(c, m)$ with $u_2 > 0$. FOC:

$$\frac{u_2(c_t, m_t)}{u_1(c_t, m_t)} = \frac{i_t}{1 + i_t}$$

Money demand decreasing in the opportunity cost $i/(1 + i)$.

Transactions costs: Consumption requires $c(1 + s(m/c))$ units of the good, with $s' < 0$. Higher real balances reduce transaction costs.

Common Features of the Reduced-Form Models

All three reduced-form models (CIA, MIU, transactions costs) share:

- Rate-of-return dominance: $i > 0 \Rightarrow$ bonds dominate money
- Friedman rule ($i = 0$) is optimal
- Quantity theory in some form

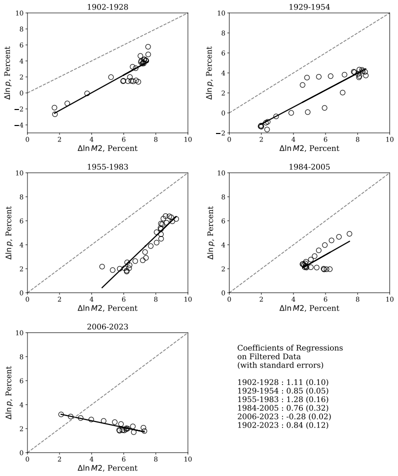
The Quantity Theory in the Data

Stable money demand and proportionality between M and PY are the defining features of the QTM. Inflation should move one-for-one with money growth in excess of real GDP growth.

Both series are filtered as 15-year symmetric moving averages, and the sample is split into five subperiods: 1902–1928, 1929–1954, 1955–1983, 1984–2005, 2006–2023.

Inflation and Money Growth

Filtered inflation against filtered M2 growth, by subsample



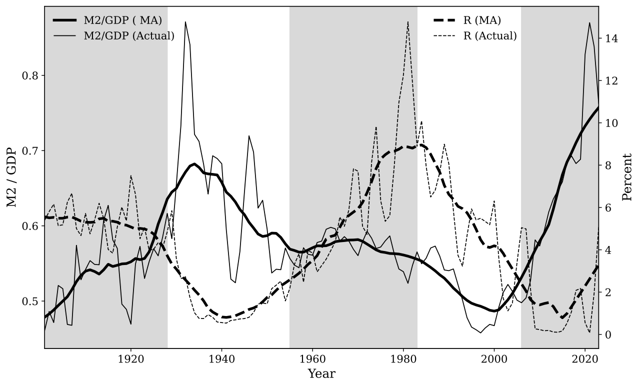
Inflation and Money Growth: Reading the Figure

Circles are the filtered data; the dashed line is 45 degrees and the thick line is the OLS fit of inflation on money growth.

- Full sample 1902–2023: slope 0.84 (0.12) — close to, but below, one-for-one
- Subsample slopes range from 1.28 (1955–1983) to -0.28 (2006–2023)
- Standard errors are corrected for heteroskedasticity and autocorrelation

Money Demand

M2/GDP ratio and the 6-month commercial paper rate



Thin lines: annual data. Thick lines: 15-year symmetric moving averages. Shading separates the five subsamples.

Money Demand: Is the Relationship Stable?

- On the filtered data the relationship looks stable: whenever the filtered short rate rises, the filtered M2/GDP ratio falls
- That stability **disappears in the subsamples**: the OLS coefficient of M2/GDP on the short rate can be positive or negative, and is usually insignificant

A cautious summary is that the evidence on the QTM is **mixed**.

The Fisher Equation

$$1 + i_t = (1 + r_t)(1 + \pi_t)$$

Gross nominal rate $=$ gross real rate $\times$ gross inflation rate. In perfect foresight, this is an arbitrage condition between real and nominal bonds.

Defines the price level $P = 1/p_m$ and inflation $1 + \pi = P'/P$.

Policy and Indeterminacy

Hyperinflation Equilibria

In the CIA model with constant money growth $\gamma > 1$, combining FOCs:

$$\gamma m_t \, u_2(m_t, 1 - m_t) = \beta m_{t+1} \, u_1(m_{t+1}, 1 - m_{t+1})$$

Two steady states: one with $m > 0$ and one with $m = 0$. Under some conditions on u, for any $m_0 < \bar{m}$, real balances converge to zero: money steadily loses value faster than money is printed $\Rightarrow$ **hyperinflation**.

Interest Rate Rules and the Taylor Principle

A pure interest-rate peg leaves the initial price level P_0 indeterminate (real indeterminacy, not just nominal).

Interest rate rule (Taylor, 1993):

$$i_t = \phi\left(\frac{P_t}{\tilde{P}_t}\right)$$

where $\tilde{P}_t$ is a target price level.

The Taylor Principle

Linearizing around steady state: local dynamics of $P_t/\tilde{P}_t$ governed by $\phi'(1)/(1+\tilde{i})$.

- $\phi'(1) > 0$ (**Taylor principle**): unique bounded equilibrium $P_t = \tilde{P}_t$
- $\phi'(1) \leq 0$: indeterminacy (continuum of converging paths)

Raising the nominal rate *more than one-for-one* in response to price level deviations eliminates indeterminacy.

The Cashless Limit

MIU with CES aggregator: $h(c, m_1) = [(1-\omega)c^\rho + \omega m_1^\rho]^{1/\rho}$, $\rho < 1$.

Money demand:

$$\frac{M_{t+1}}{P_t} = c_t \left(\frac{1-\omega}{\omega} \cdot \frac{i_t}{1+i_t} \right)^{\frac{1}{\rho-1}}$$

As $\omega \to 0$: $M/P \to 0$ but the elasticities of money demand remain unchanged. If P_t is targeted via an interest rate rule, $M_{t+1} \to 0$.

The Cashless Limit: Implications

The linearized Euler equation no longer involves money at all $\Rightarrow$ the real side and the price level are determined without money.

This motivates the **New Keynesian model** (Chapter 18): nominal variables without a money stock. The interest rate rule is essential for the cashless limit to work.

Government Budget and Seigniorage

Consolidated government flow budget:

$$g_t + (1 + i_{t-1})\frac{B_t}{P_t} = \tau_t + \frac{B_{t+1}}{P_t} + \frac{M_{t+1} - M_t}{P_t}$$

Seigniorage: $(M_{t+1} - M_t)/P_t$ — revenue from printing money.

Seigniorage in Present Value

In present value:

$$\sum_{t=0}^{\infty} q_{0,t}(g_t - \tau_t) + (1 + i_{-1})\frac{B_0}{P_0} = \sum_{t=0}^{\infty} q_{0,t}\frac{M_{t+1} - M_t}{P_t}$$

Primary deficit + initial debt = present value of seigniorage.

Sargent-Wallace ("Unpleasant Monetarist Arithmetic"): Under fiscal dominance, open-market operations to fight inflation today raise debt and require more seigniorage later $\Rightarrow$ higher long-run inflation.

Fiscal Theory of the Price Level

Suppose the government commits to a tax rule:

$$\tau_t - \tilde{\tau} = -\frac{M_{t+1} - M_t}{P_t}$$

Substituting into the present-value budget:

$$(1 + i_{-1})\frac{B_0}{P_0} = \frac{\tilde{\tau} - \tilde{g}}{1 - \beta}$$

The only endogenous variable is $P_0 \Rightarrow$ **unique price level**. The commitment to future fiscal adjustments pins down the value of initial government liabilities.

Irrelevance of Open-Market Operations

Irrelevance of open-market operations (Wallace, 1981): When $i_t = 0$, money and bonds are perfect substitutes. Swapping M for B (holding $M + B$ fixed) has no real effects.

Multiple Currencies and Exchange Rates

Kareken-Wallace Exchange Rate Indeterminacy

OLG with two currencies a and b. No short-selling. Both valued only if they earn the same return. Define nominal exchange rate $e_t \equiv p_{a,t}/p_{b,t}$.

If returns are equalized: $e_t = e$ (constant). Total money in b units: $M = eM_a + M_b$. The consumer's problem collapses to a single-currency problem with M.

Kareken-Wallace: Indeterminacy and Gresham's Law

Result: e is **indeterminate**. Any $e \in (0, \infty)$ is consistent with equilibrium. Fiat currencies are perfect substitutes as stores of value.

With different money growth rates ($\gamma_a > \gamma_b$): the fast-growing currency dominates the total money stock in the long run (**Gresham's law**: "bad money drives out good").

Reduced-Form Models and Crypto-Currency

Lucas (1982): Two countries, two goods, CIA constraint per good. Good 1 requires country 1 money $\Rightarrow$ exchange rate tied to relative demands $\Rightarrow$ uniquely determined.

Crypto-currency: No intrinsic value; can serve as store of value or medium of exchange.

- OLG perspective: just another currency, indeterminate value, random fluctuations possible
- CIA/MIU: can be modeled as an additional means of payment, but results follow directly from assumptions

Money as Store of Value with Missing Assets

Missing Assets: The Huggett Model with Money

Dynastic model with idiosyncratic income risk and tight borrowing constraint ($a' \geq 0$). Without fiat money: autarky equilibrium. Gross interest rate:

$$\frac{1}{q} = \frac{1}{\beta} \cdot \frac{1}{\pi_{hi|hi} + \pi_{lo|hi} \frac{u'(\omega_{lo})}{u'(\omega_{hi})}} < \frac{1}{\beta}$$

If $1/q < 1$ (negative net rate): agents want to save but cannot.

Huggett with Money: Equilibrium

Introduce fiat money with constant P: money serves as riskless asset with $q = 1$. Total real money stock = total savings:

$$m = \int g_\omega(a)\,\Gamma(d\omega, da)$$

This pins down P. Similar to Townsend (1980) turnpike model. Money has value because other assets are missing—but it cannot explain rate-of-return dominance.

Money as a Medium of Exchange

Kiyotaki-Wright (1989)

Three types of agents; type i consumes good $i + 1$ (mod 3) and produces good i. **Never a double coincidence of wants**. Goods are indivisible and storable at cost c_i.

Random pairwise meetings. A good can be accepted and stored to trade later $\Rightarrow$ functions as a medium of exchange. Nash equilibrium: trades only occur if mutually voluntary.

Kiyotaki-Wright: Money as Medium of Exchange

Fiat money: Intrinsically useless, indivisible, storable at low cost $c_{\$}$. Under some parameter restrictions, money is accepted in trade $\Rightarrow$ **valued as medium of exchange**. Another equilibrium: money never accepted, never valued.

Lagos and Wright (2005): Generalize to divisible money and goods; decentralized + centralized markets. Tractable due to quasi-linear preferences. Modern workhorse for monetary search theory.

Summary

Key Takeaways (I)

1. **OLG**: Money can have value as a store of value when $\omega_y > \omega_o$. Continuum of equilibria; only the stationary one is Pareto efficient. Number of nominal units irrelevant.
2. **Dynastic models**: Fiat money cannot have value as a pure store of value (TVC argument). Need reduced-form liquidity services (CIA, MIU, transactions costs) to give money value.
3. **CIA model**: Binding CIA acts like a tax on labor. Steady-state distortion: MRS $= \beta z < z =$ MRT. Quantity theory: $M = Pc$.
4. **Friedman rule**: Shrink money at rate β (or pay interest on money) $\Rightarrow i = 0 \Rightarrow$ eliminates distortion.
5. **Fisher equation**: $1 + i = (1 + r)(1 + \pi)$.

Key Takeaways (II)

6. **Indeterminacy**: Common in monetary models. Hyperinflation paths exist under constant money growth. Taylor principle ($\phi' > 0$) restores uniqueness under interest rate rules.
7. **Cashless limit**: As money's weight $\omega \to 0$ in MIU, $M/P \to 0$ but the price level remains determinate under a Taylor rule. Motivates the New Keynesian model.

Key Takeaways (III)

7. **Fiscal interactions**: Sargent-Wallace (fiscal dominance $\Rightarrow$ fighting inflation is hard); FTPL (fiscal commitment pins down P_0); Wallace irrelevance (at $i = 0$, OMOs have no effect).
8. **Multiple currencies**: Kareken-Wallace exchange rate indeterminacy (pure store of value); CIA models deliver unique exchange rates. Crypto: same theoretical logic.
9. **Fundamental theories**: Missing assets (Huggett/Townsend: money fills the role of absent riskless asset). Medium of exchange (Kiyotaki-Wright: search frictions + no double coincidence $\Rightarrow$ valued money endogenously).