Chapter 25: Sustainability

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Outline

Introduction: The Economy and the Environment

Why Climate Change Is a Macro Topic

- Climate change is **global**: caused by aggregate CO_2 emissions, a byproduct of fossil energy — our dominant energy source
- Energy services cost roughly **5% of global GDP**; sharp energy-use reductions can cause recessions (1970s oil shocks)
- Climate change is a **long-run** issue with no direct evidence on long-run policy effects $\Rightarrow$ macroeconomic *modeling* is the laboratory
- Minor departures from the standard growth model suffice

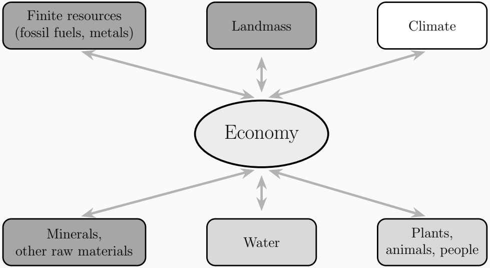
Why Climate Change Is a Macro Topic (cont.)

Broader **sustainability** questions: do present and future humans have enough? Should we economize on natural resources? Is "de-growth" warranted?

Chapter plan: economy vs. environment; natural-science background; Integrated Assessment Models (IAMs); finite natural resources (Hotelling, directed technical change).

The Economy and the Environment

The economy and the environment — landmass, finite resources, climate, water, plants/animals/people, minerals; darker shades = higher degree of property rights

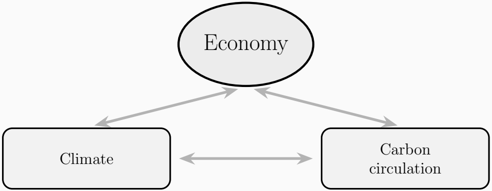


The Economy and the Environment (cont.)

- **Landmass**: highest degree of property rights
- **Finite resources** (fossils, minerals): largely owned — Coasean management by owners is possible, so overconsumption is not obvious at the outset
- **Climate**: *no* property rights possible — emissions mix globally within days; the concentration is the same everywhere
- Water, biodiversity, land quality: endogenous to human activity and to the climate

The Economy and the Environment (cont.)

IAMs consist of three interacting blocks: economy $\rightarrow$ carbon circulation $\rightarrow$ climate $\rightarrow$ (damages) economy



Natural-Science Background

The Greenhouse Effect and the Energy Balance

CO_2 does not affect incoming solar radiation but **obstructs outgoing heat radiation**: a CO_2 increase creates an energy-balance surplus and the temperature rises (the **greenhouse effect**). Without greenhouse gases: $-20°$C instead of $+15°$C.

Let $T =$ surface temperature deviation from pre-industrial. With forcing f (reduced outflow), heat radiation proportional to T (coefficient κ, embodying uncertain feedbacks: ice-albedo, clouds):

$$\frac{dT(t)}{dt} = \sigma\left(f(t) - \kappa T(t)\right)$$

Steady state after constant forcing f: $T = f/\kappa$.

The Greenhouse Effect and the Energy Balance (cont.)

Arrhenius greenhouse law (1896): forcing is logarithmic in the atmospheric carbon stock S relative to pre-industrial S_0:

$$f_{CO_2} = \frac{\eta}{\log 2} \log \frac{S}{S_0}, \qquad \eta \approx 3.7$$

CO_2: about two thirds of greenhouse-gas forcing; up $\sim$50% since the Industrial Revolution.

Combining the steady state with the Arrhenius law:

$$T(f(S)) = \frac{\eta}{\kappa} \frac{1}{\log 2} \log \left(\frac{S}{S_0} \right)$$

η/κ is the **equilibrium climate sensitivity (ECS)**: the long-run warming from a *doubling* of CO_2. IPCC 6th report: "likely" between $2.5°$C and $4°$C, best estimate $3°$C (90% interval 2–$5°$C).

Climate Sensitivity and Ocean Dynamics (cont.)

Two-layer system with oceans (ocean temperature T^L, slow to warm):

$$T_t = T_{t-1} + \sigma_1 \left[\frac{\eta}{\log 2} \log \left(\frac{S_{t-1}}{S_0} \right) - \kappa T_{t-1} - \sigma_2 \left(T_{t-1} - T^L_{t-1} \right) \right]$$

$$T^L_t = T^L_{t-1} + \sigma_3 \left(T_{t-1} - T^L_{t-1} \right)$$

Same steady state, but $\sigma_3 \ll \sigma_1$ makes convergence much slower — the ocean exerts a long-lasting (but not permanent) cooling drag.

The Carbon Cycle

Carbon circulates between three reservoirs: atmosphere S, surface oceans/biosphere S^U, deep oceans S^L. Linear system (Nordhaus RICE):

$$S_t - S_{t-1} = \phi_{12} S_{t-1} + \phi_{21} S^U_{t-1} + E_{t-1}$$

$$S^U_t - S^U_{t-1} = \phi_{12} S_{t-1} - (\phi_{21} + \phi_{23}) S^U_{t-1} + \phi_{32} S^L_{t-1}$$

$$S^L_t - S^L_{t-1} = \phi_{23} S^U_{t-1} + \phi_{32} S^L_{t-1}$$

Folini et al. (2021): closely replicates CMIP5 Earth System Models with $\phi_{12} = 0.053$, $\phi_{21} = 0.0536$, $\phi_{23} = 0.0042$, $\phi_{32} = 0.001422$ (annual). Units: 1 kg carbon $\approx$ 3.67 kg CO_2.

The Carbon Cycle (cont.)

Reduced form (Golosov-Hassler-Krusell-Tsyvinski, 2014) for excess atmospheric carbon:

$$S_t - S_0 = \sum_{s=0}^{\infty} (1 - d_s) E_{t-s}, \qquad 1 - d_s = \phi_L + (1 - \phi_L)\phi_0(1 - \phi)^s$$

Matching IPCC/Archer: ∼20–25% of emissions stay for millennia ($\phi_L = 0.2$); ∼50% leave within decades ($1 - \phi_0$); the rest decays geometrically ($\phi = 0.0228$, decadal; $\phi_0 = 0.393$).

The Constant Carbon-Climate Response (CCR)

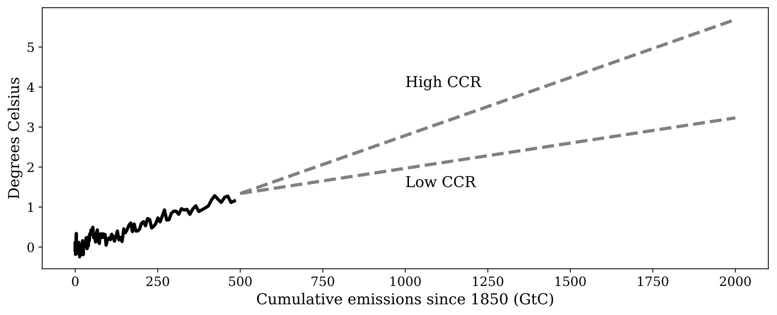
A key emergent property of the combined system: temperature is approximately **proportional to cumulative emissions**:

$$T_t \approx \sigma_{CCR} \sum_{s=0}^{t} E_s$$

IPCC 6th report: σ_{CCR} "likely" between 1.0 and 2.3°C per 1,000 GtC. Two offsetting effects: CO_2 slowly leaves the atmosphere (less forcing) while oceans slowly warm (less cooling) — balancing on a centennial scale.

The Constant Carbon-Climate Response (CCR) (cont.)

Cumulative emissions vs. global mean temperature; solid: data 1850–2022; dashed: high/low CCR forecasts



Implications of a Constant CCR

1. **Irreversibility**: temperature depends on *total past* emissions — there is effectively no depreciation in the emissions-to-temperature map. To stop warming, emissions must stop.
2. **Carbon budget**: a temperature target translates into a total emissions budget. By 2022 we had burnt ≈650 GtC; with CCR $= 1$, that is $0.65°$C of warming, leaving 850 GtC for the $1.5°$C target (≈85 years at current rates); with CCR $= 2.3$, the $1.5°$C budget is *already exhausted*.

Implications of a Constant CCR (cont.)

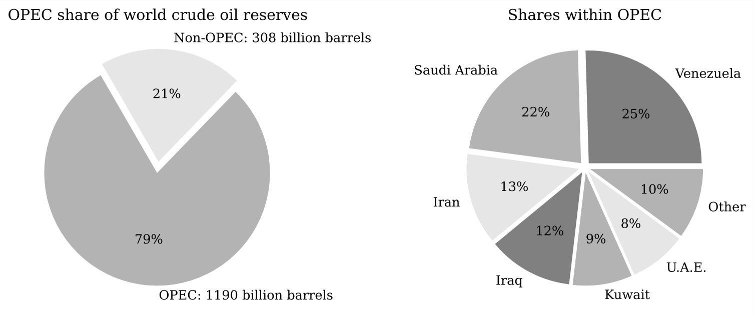
3. **Path multiplicity**: many emission paths meet a given budget — front-loaded or back-loaded reductions — but they are *not* welfare-equivalent, which a date-target alone ignores. Evaluating them requires the climate-economy interaction (the IAM below).
4. **Linearity**: a constant CCR rules out non-linearities such as **tipping points**.

The Fossil Energy Supply

Reserves: recoverable with reasonable certainty under existing conditions. **Resources**: best guess including deposits unprofitable today.

The Fossil Energy Supply (cont.)

World crude oil reserves — OPEC 1,190 billion barrels (≈80%), non-OPEC 308



Conventional oil flows on its own: marginal cost a fraction of price (Hotelling rents); **unconventional** oil costs near or above price. Expect all conventional oil to be burnt.

The Fossil Energy Supply (cont.)

Warming from burning the reserves (constant CCR, 0.27–0.63°C per 1,000 $GtCO_2$):

- Oil: 1,190bn barrels $= (1{,}190/7.33) \times 0.85 \times 3.67 = 500$ $GtCO_2$ $\Rightarrow$ **0.13–0.33°C**
- Coal: 1,074 GtC $\times 0.70 \times 3.67 = 2{,}759$ $GtCO_2$ $\Rightarrow$ **0.75–1.73°C**
- Gas: 190 trillion m^3 $\Rightarrow$ less than $0.1°$C

Reserves vs. Resources

Reserves and resources in GtC

	Reserves	Resources (Rogner)	Resources (BGR)
Oil (conv+unconv)	173	≈400–2,200	≈500
Coal	1,074	≈6,200	≈11,500
Gas	100	≈350	≈365

Sources: Rogner (1997); BGR (2020).

Reserves vs. Resources (cont.)

Coal **resources** are an order of magnitude larger than reserves (proven coal reserves are likely underestimated: coal is not very profitable, so search incentives are weak).

Burning all the coal resources implies 10°**C or more** of warming via the constant CCR. The binding constraint on warming is therefore *not* the fossil supply — it must come from policy or economics.

Damages: Externalities and Measurement Approaches

Warming effects are classical **externalities**: unpriced welfare consequences of emissions. Since emissions mix globally, the externality per unit is **location-independent** $\Rightarrow$ the Pigou prescription is a *uniform global* tax on emissions at marginal damage. (Stern Review 2007; Nordhaus.)

Bottom-up: identify all damages (agriculture, sea level, health, amenities, catastrophes), value them, add up $\rightarrow$ a damage function $D(T)$. Nordhaus: modest aggregates — 2°C costs under 1% of global GDP. PESETA (EU): similar magnitudes; the north may gain, the Mediterranean loses.

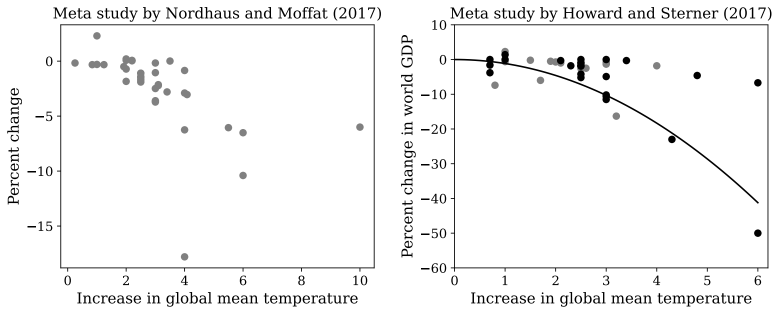
Damages: Externalities and Measurement Approaches (cont.)

Top-down: relate aggregates to climate variation.

- Time series: $+1^{\circ}$C $\Rightarrow$ -1pp output *growth* in poor countries, none in rich (Dell-Jones-Olken); $+4^{\circ}$C $\Rightarrow$ global GDP -7% by 2100 (Kahn et al.); mortality cost of 5°C scenario: 3.2% of GDP (Carleton et al.)
- Cross section: strong negative output-temperature correlation; U-shaped productivity with a “sweet spot” around 11°C annual average (Cruz–Rossi-Hansberg; Krusell-Smith)

Damage Functions, Adaptation, Discounting

Meta studies of climate damages: Nordhaus–Moffat (2017) and Howard–Sterner (2017)



Low average costs at 1–2°C; convex in temperature.

Damage Functions, Adaptation, Discounting (cont.)

Nordhaus damage function (share of GDP lost):

$$D(T) = 1 - \frac{1}{1 + 0.00267T^2} \approx 0.024 \left(\frac{T}{3}\right)^2$$

i.e., 2.4% of output lost at $3°$C. **GHKT**: mapping $S \to T \to D$ is well approximated by the TFP factor $e^{-\gamma S}$ — damages exponential in atmospheric carbon directly.

Damage Functions, Adaptation, Discounting (cont.)

Adaptation: with damages $(T-a)^x/x+a$ and linear adaptation costs, optimizing over a yields a *linear* reduced-form $D(T)=T+(1-x)/x$ regardless of x: adaptation flattens convexity.

Discounting: revealed preference (Euler equation, interest rates) gives $\beta \approx 0.985$ (Nordhaus); the Stern Review argues for $\beta \approx 0.999$ on ethical grounds — the choice matters enormously for largely irreversible damages, but is philosophy more than economics.

Integrated Assessment Models

A Static One-Region IAM

Three competitive sectors; energy services one-for-one from coal at constant marginal cost p. With $K = L = 1$:

$$Y = AE^{\nu}$$

Private economy with coal tax τ:

$$\pi_e = \max_E AE^{\nu} - (p+\tau)E \qquad \Rightarrow \qquad \nu AE^{\nu-1} = p + \tau$$

so $E = [\nu A/(p+\tau)]^{1/(1-\nu)}$: coal use decreasing in the tax.

A Static One-Region IAM (cont.)

Damages (from the $E \to S \to T$ mapping, quadratic):
$D(E) = \gamma E^2$, additively separable:

$$C = AE^{\nu} - pE - \gamma E^2$$

Planner: $\nu A E^{\nu-1} = p + 2\gamma E$. Laissez-faire ($\tau = 0$) overuses coal whenever $\gamma > 0$.

Pigou tax: $\tau = 2\gamma E^*$ implements the first best — tax the externality at marginal damage. Near $\tau = 0$ the private objective is flat (smooth maximum): a modest tax has **negligible private cost but potentially large welfare benefits**.

Prices vs. Quantities

In practice politicians often prefer **quantity restrictions**: e.g., the EU Emission Trading System (cap-and-trade, 2005). Each unit of coal burnt requires an **emission right**; rights (price λ) handed to the coal producer and traded.

Prices vs. Quantities (cont.)

With cap $E \leq E^*$, the final-good firm's FOC is:

$$\nu A E^{\nu-1} = p + \lambda$$

$\lambda = \tau = 2\gamma E^*$ clears the market: **price and quantity regulation are equivalent** (Weitzman: equivalence can break under uncertainty). A cap above laissez-faire use: $\lambda = 0$; a tighter cap: λ read off the FOC.

Firm heterogeneity ($\nu_1 \neq \nu_2$): the planner's FOCs are $\nu_i A E_i^{\nu_i - 1} = 2\gamma(E_1 + E_2)$. A single tax $\tau = 2\gamma(E_1^* + E_2^*)$ — or a single *total* cap with tradable rights — delivers the optimum: facing the same price, firms equate marginal products, allocating energy efficiently. No firm-specific quantities needed: exactly how the EU ETS works.

A Dynamic IAM (GHKT): Setup

Two regions: an **oil producer** (endowed with finite conventional oil R, costless extraction) and an **oil consumer**. Log utility $\sum_{t=0}^{\infty} \beta^t \log(C_t)$ in each region.

Production with damages (S from the GHKT carbon cycle):

$$Y_t = \underbrace{\exp(z_t - \gamma S_t)}_{\equiv A_t} L_t^{1-\alpha-\nu} K_t^{\alpha} E_t^{\nu}$$

A Dynamic IAM (GHKT): Setup (cont.)

Energy services (CES over oil, coal, green; coal and green produced linearly at costs p_c, p_g):

$$E_t = \left[\lambda_o e_{o,t}^{\rho} + \lambda_c e_{c,t}^{\rho} + \lambda_g e_{g,t}^{\rho}\right]^{1/\rho}, \qquad \lambda_o + \lambda_c + \lambda_g = 1$$

Resource constraint (full depreciation; period = 10 years):

$$C_t + K_{t+1} = A_t L_t^{1-\alpha-\nu} K_t^{\alpha} E_t^{\nu} - p_{o,t} e_{o,t} - p_{c,t} e_{c,t} - p_{g,t} e_{g,t}$$

Emissions $M_t = e_{o,t} + e_{c,t}$; oil stock $R_{t+1} = R_t - e_{o,t} \geq 0$. Carbon tax τ_t rebated lump-sum within the period.

Dynamic IAM: Closed-Form Equilibrium

Oil supplier: maximizing $\sum_{t=0}^{\infty} \beta^t \log(C_{o,t})$ with $C_{o,t} = p_{o,t} e_{o,t}$ gives

$$e_{o,t} = (1-\beta) R_t \qquad \Rightarrow \qquad R_{t+1} = \beta R_t$$

Energy demands (CES cost minimization, price index P_t):

$$e_{\kappa,t} = E_t \left(\frac{P_t \lambda_\kappa}{p_{\kappa,t}} \right)^{\frac{1}{1-\rho}}, \quad \kappa \in \{o, c, g\}$$

$$E_t = \left(\nu \frac{A_t L_t^{1-\alpha-\nu} K_t^{\alpha}}{P_t} \right)^{\frac{1}{1-\nu}}$$

Dynamic IAM: Closed-Form Equilibrium (cont.)

Constant saving rate: with log utility, Cobb-Douglas, full depreciation, and rebates, the Euler equation delivers

$$s_t = \frac{\alpha\beta}{1-\nu} \equiv s \quad \forall t, \qquad K_{t+1} = s(1+\Gamma_t)\hat{Y}_t$$

The allocation is determined **sequentially with no forward-looking terms**: only the oil price needs solving each period (supply is predetermined at $(1-\beta)R_t$) — the model solves in under a second even with many regions.

The Optimal Carbon Tax

The marginal damage externality of energy source κ:

$$\Lambda_t^{\kappa} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{U'(C_{t+j})}{U'(C_t)} \frac{\partial F_{t+j}}{\partial S_{t+j}} \frac{\partial S_{t+j}}{\partial e_{\kappa,t}}$$

With $e^{-\gamma S}$ damages, $\frac{\partial F_t}{\partial S_t}\frac{1}{Y_t} = -\gamma$ (constant *percentage* loss per carbon unit); log utility gives $U'(C) = 1/C$; the carbon cycle gives $\partial S_{t+j}/\partial e_{\kappa,t} = 1 - d_j$. The **optimal (Pigou) tax** becomes:

$$\tau_t = Y_t \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j \gamma (1 - d_j) \right]$$

The Optimal Carbon Tax (cont.)

Proportional to GDP, and depending only on **three "d's"**:

1. **depreciation** of atmospheric carbon (d_j): the longer carbon stays, the higher the tax
2. **damages** (γ)
3. **discounting** (β): more weight on the future, higher tax

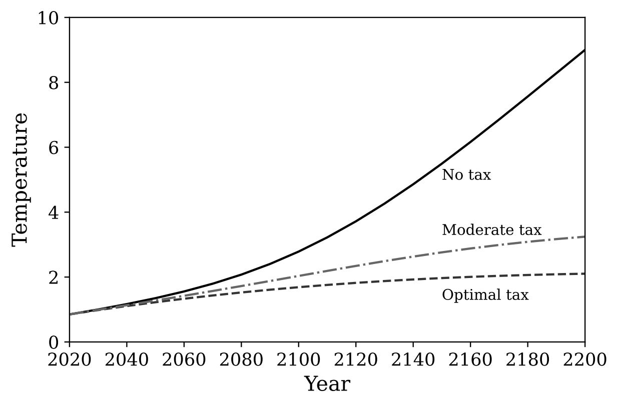
No other parameter — not even how fossil fuel is produced — enters.

Simulation: The Power of the Carbon Tax

Calibration follows GHKT and Hassler et al.: standard long-run targets, energy parameters from observed prices and shares, energy substitution elasticity 2, Nordhaus damages. Optimal tax $\approx$ **US\$25 per ton $\mathbf{CO_2}$** ($\approx$ \$90/tC); moderate tax = one third $\approx$ \$8/ton.

Simulation: The Power of the Carbon Tax (cont.)

Global temperature under no tax, moderate tax, optimal tax, 2020–2200



Simulation: The Power of the Carbon Tax (cont.)

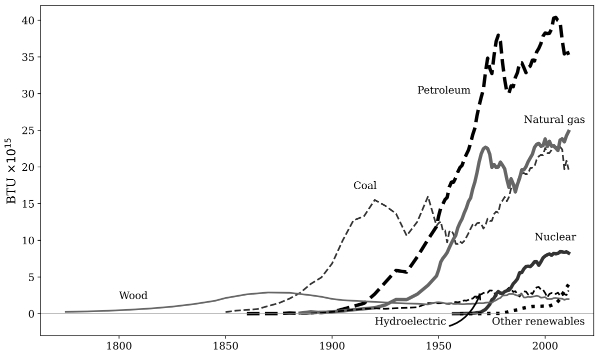
- **No tax**: $+9°$C by 2200
- **Optimal tax**: contained around $+2°$C
- Even the **moderate** tax achieves a large share of the containment

The carbon tax is a remarkably potent mitigation instrument; the sensitivity of the $25 figure to the three "d's" (especially discounting) should be kept in mind.

Natural Resources in Finite Supply

Data: Quantities and Prices

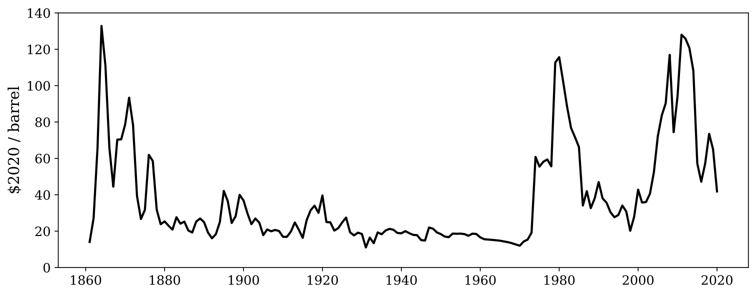
US energy consumption by source, 1800–2020 — strong upward trends; potential peaks for oil and coal



Per capita, oil use peaked in the 1970s in the US and the world.

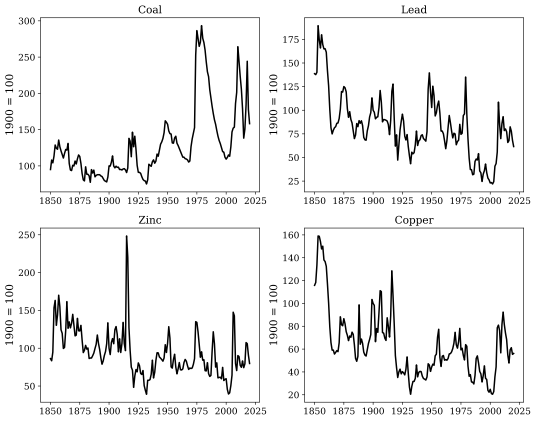
Data: Quantities and Prices (cont.)

Crude oil price, 1860–2020 (2020 dollars) — enormous volatility; no long-run trend, possibly positive postwar trend



Data: Quantities and Prices (cont.)

Inflation-adjusted prices of coal, lead, zinc, copper — stationary; metals show no postwar upward trend



Data: Quantities and Prices (cont.)

Summary: price swings of stock-market magnitude with weak trends; quantities rising strongly, with fossil-fuel slowdowns late in the sample. A challenge for the theory.

Cake Eating and the Hotelling Rule

Planner (Hotelling 1931; Gale 1967): $\max \sum_{t=0}^{\infty} \beta^t \log c_t$ subject to $\sum_{t=0}^{\infty} c_t = R_0$:

$$c_t = (1-\beta)R_t = (1-\beta)\beta^t R_0$$

Utility is bounded despite $c_t \to 0$; consumption is front-loaded purely by discounting.

Cake Eating and the Hotelling Rule (cont.)

Market outcome: Arrow-Debreu prices satisfy $\hat{p}^o_t = \hat{p}^o_{t+1}$ (zero marginal cost $\Rightarrow$ indifference about when to sell) $\Rightarrow$ market replicates the planner. In **spot** terms:

$$p^o_t = \frac{p_{t+1}}{p_t} p^o_{t+1},$$

i.e. the resource price grows at the real interest rate — the **Hotelling rule**.

Cake Eating and the Hotelling Rule (cont.)

Hotelling rents: $p^o_t > 0$ despite zero production cost and perfect competition: scarcity itself commands a rent.

With extraction costs: indifference requires $p^o_t - mc_t = (p^o_{t+1} - mc_{t+1})/(1 + r_t)$ — the *net* price grows at the interest rate; mc is a broad concept including dynamic depletion effects.

The Dasgupta-Heal Production Economy

The resource as a production input (Cobb-Douglas, $\delta = 1$, fixed labor):

$$\max \sum_{t=0}^{\infty} \beta^t \log c_t \quad \text{s.t.} \quad c_t + k_{t+1} = A z_t k_t^{\alpha} e_t^{\nu}, \qquad \sum_{t=0}^{\infty} e_t = R$$

Cobb-Douglas justified by roughly trendless (if volatile) energy cost shares in the long run.

The Dasgupta-Heal Production Economy (cont.)

Solution: extraction as in cake eating, $e_t = (1-\beta)\beta^t R_0$; saving $k_{t+1} = \alpha\beta A z_t k_t^{\alpha} e_t^{\nu}$; long-run gross growth with z growing at γ_z:

$$g = (\gamma_z \beta^{\nu})^{\frac{1}{1-\alpha}}$$

Technology growth can deliver positive output growth *despite* resource depletion ($g > 1$ for γ_z large enough).

Prices: $1 + r = g/\beta$ and $p_t = \nu y_t / e_t$ grows at $g/\beta = 1 + r$: the Hotelling rule again. With $r > 0$, the resource price should rise *exponentially* — hard to see in postwar data, though unobservable marginal costs and the choice of interest rate complicate the test.

Capital-Energy Complementarity

The simple model cannot explain the strong **upward trend in resource use**. Alternative: in the short run, energy is highly *complementary* with capital and labor. Extreme case (Leontief):

$$y_t = \min\{Ak_t^\alpha, e_t\}$$

At the Industrial Revolution: $k_0 \approx 0$, A low, R_0 huge relative to Ak_0^α. Optimal path: accumulate capital, with $e_t = Ak_t^\alpha$ **rising** for a long time — then eventually declining as finiteness binds. Matches the rise-then-peak pattern in the data.

Capital-Energy Complementarity (cont.)

General CES:

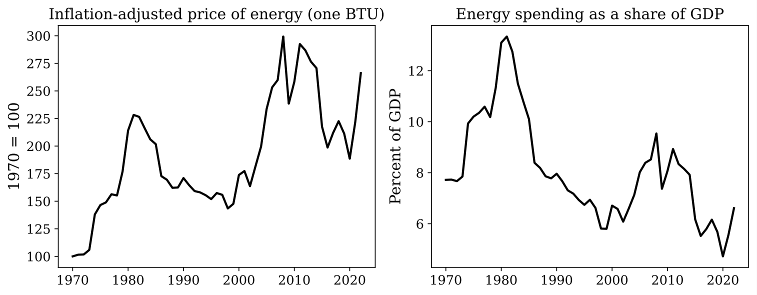
$$y_t = \left[(1-\nu) \left(A_t k_t^{\alpha} l_t^{1-\alpha} \right)^{\frac{\varepsilon-1}{\varepsilon}} + \nu \left(A_{e,t} e_t \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Given α, ν, ε, the factor-specific technologies can be **backed out** from FOCs and observed cost shares:

$$A_t = \frac{y_t}{k_t^{\alpha} l_t^{1-\alpha}} \left(\frac{l_t^{share}}{1-\alpha} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \qquad A_{e,t} = \frac{y_t}{e_t} \left(e_t^{share} \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Evidence: Directed, Endogenous Technical Change

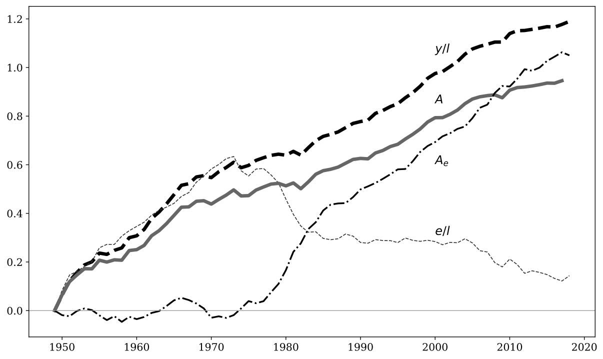
Price of fossil energy and its cost share, US 1970–2020 — the cost share tracks the price closely



A rising price raising the cost *share* implies $\varepsilon \approx 0$ (near-Leontief): with Cobb-Douglas ($\varepsilon = 1$) the share would be constant; with $\varepsilon > 1$ it would fall.

Evidence: Directed, Endogenous Technical Change (cont.)

US output per worker, energy per worker, and backed-out A and A_e, 1949–2020



Evidence: Directed, Endogenous Technical Change (cont.)

- 1949–1973: e/l tracks y/l; energy-saving technical change A_e **flat**
- Post-1973 (oil shocks): A_e accelerates sharply while e/l falls; energy in *efficiency units* $A_e e/l$ keeps rising; capital/labor-augmenting A slows

$\Rightarrow$ technology is **endogenous and directed**: innovation shifts toward saving on the input that became expensive (cf. Chapter 13; Romer variety or Aghion-Howitt quality interpretations of (A, A_e)).

The Technology Menu and Long-Run Substitutability

Hassler et al. (2021): firms freely pick next-period factor-saving technologies from a **menu**

$$G\left(\frac{A_{t+1}}{A_t}, \frac{A_{e,t+1}}{A_{e,t}}\right) = 0$$

with G increasing in both arguments (a trade-off frontier), taking economy-wide averages as given.

The Technology Menu and Long-Run Substitutability (cont.)

Key result: if production is CES with short-run elasticity below 1 and G is CES, the *reduced-form* production function (after optimal technology choice) is CES with a **higher elasticity of substitution** (León-Ledesma and Satchi, 2019).

$\Rightarrow$ Substitutability is low in the short run, high in the long run: a sharp rise in the energy cost share after a price hike **erodes over time** as technology redirects — fossil fuel is hard to replace today, much less so over decades.

Taking Stock on Finite Resources

- The data show rising resource use with late-sample slowdowns — qualitatively consistent with depletion theory *plus* time-varying, input-specific technology
- The wildly volatile, weakly-trending **prices** remain a challenge to explain quantitatively
- With **property rights**, the basic market outcome is efficient: owners internalize scarcity via Hotelling rents — no obvious case for intervention (beyond standard R&D externalities under endogenous directed technical change)
- This contrasts sharply with the **climate**, where property rights are impossible and a Pigou tax is required

Summary

Key Takeaways (I)

1. **IAMs** integrate three blocks: economy, carbon cycle, climate. Climate is the environmental category with *no* property rights — emissions are a global externality.
2. **Climate science**: energy balance $dT/dt = \sigma(f - \kappa T)$; Arrhenius forcing $f = \frac{\eta}{\log 2}\log(S/S_0)$; ECS $= \eta/\kappa \approx 3°$C per CO_2 doubling; two-layer ocean system slows convergence; three-reservoir linear carbon cycle; GHKT reduced form $1 - d_s = \phi_L + (1-\phi_L)\phi_0(1-\phi)^s$ (20% of emissions effectively permanent).

Key Takeaways (II)

3. **Constant CCR**: $T_t \approx \sigma_{CCR} \sum_s E_s$, 1.0–2.3°C per 1,000 GtC $\Rightarrow$ irreversibility (stopping warming requires stopping emissions), carbon budgets (650 GtC spent; the 1.5°C budget may already be exhausted at high CCR), and no tipping points by construction.
4. **Fossil supply**: conventional oil (≈0.13–0.33°C if all burnt) and gas are small; **coal** dominates (reserves: 0.75–1.73°C; resources: $10°$C$+$). Supply does not constrain warming.
5. **Damages**: bottom-up (Nordhaus: $<$1% of GDP at 2°C) and top-down (growth effects in poor countries; 11°C productivity sweet spot); $D(T) = 1 - 1/(1 + 0.00267T^2)$; GHKT $e^{-\gamma S}$; adaptation linearizes convex damages; discounting (β 0.985 vs. 0.999) is decisive and partly philosophical.

Key Takeaways (III)

6. **Static IAM**: market FOC $\nu A E^{\nu-1} = p + \tau$ vs. planner $\nu A E^{\nu-1} = p + 2\gamma E$; Pigou tax $\tau = 2\gamma E^*$; near-zero private cost of a modest tax; **prices and quantities are equivalent** (cap-and-trade $\lambda = \tau$); one total cap suffices under heterogeneity (EU ETS logic).
7. **Dynamic GHKT IAM**: log utility + Cobb-Douglas + full depreciation ⇒ closed forms — $e_{o,t} = (1-\beta)R_t$, saving rate $s = \alpha\beta/(1-\nu)$, sequential solution; optimal tax $\tau_t = Y_t \mathbb{E}\left[\sum_{j=0}^{\infty} \beta^j \gamma (1 - d_j)\right]$ — only the three "d's" matter. Calibrated: ≈\$25/tCO_2; no tax ⇒ $+9^\circ$C by 2200, optimal tax ⇒ $\approx +2^\circ$C.

Key Takeaways (IV)

3. **Hotelling**: cake eating gives $c_t = (1-\beta)\beta^t R_0$; resource prices (net of marginal cost) grow at the real interest rate; scarcity rents exist under perfect competition; Dasgupta-Heal: $g = (\gamma_z \beta^\nu)^{1/(1-\alpha)}$ — growth despite depletion; exponential price growth is hard to see in the data.

Key Takeaways (V)

2. **Complementarity and directed technology**: near-Leontief short-run production (cost share tracks price) explains rising-then-peaking resource use; post-1973 acceleration of energy-saving A_e shows technology is endogenous and **directed**; the technology menu raises long-run substitutability above short-run.
3. **Policy contrast**: the climate requires intervention (Pigou tax) because property rights are impossible; owned finite resources are managed efficiently by markets under ideal conditions.